BOHR International Journal of Engineering
2022, Vol. 1, No. 1, pp. 1–8
DOI: 10.54646/bije.2022.01
www.bohrpub.com

RESEARCH PAPER

Tire road surface conditions and 2D curve analysis based development of surrogate safety measures

Pushkin Kachroo[1*] **and K. Ramachandra Rao**[2]

[1]Department of Electrical and Computer Engineering, University of Nevada Las Vegas (UNLV), Las Vegas, NV, United States
[2]Department of Civil Engineering, Indian Institute of Technology Delhi, New Delhi, India

[*]**Correspondence:**
Pushkin Kachroo,
pushkin@unlv.edu

Received: 03 June 2022; **Accepted:** 18 March 2022; **Published:** 05 July 2022

This paper presents a new comprehensive way to perform safety surrogate measure analysis that uses longitudinal as well as lateral dynamics for vehicle trajectory analysis as compared to traditional methods that use only longitudinal predicted movement for crashes. The paper also introduces, for the first time, longitudinal and lateral adhesion coefficients in the dynamics to account for the interaction between the tires and the road surface that accounts for many road accidents.

Keywords: safety, surrogate, vehicle dynamics, lateral dynamics, adhesion, wheel slip

Introduction

Background

Traffic accidents are rare compared to the traffic that is prevalent most of the time (1, 2). For instance, if we con- sider a single road and consider traffic on it from time t_1 to t_2 and divide the time interval into small time segments of length Δt, we will be able to see that the percentage of intervals that have accidents on them compared to the total number of accidents will be a very small number.

Traffic being a rare event phenomenon makes it very difficult to collect data on it and use it to build models. For traffic accidents, we can use surrogates for accidents, where the frequency of the surrogates is much higher. We can use traffic conflicts as safety surrogate measures (SSMs) for traffic accidents. Some of the SSMs developed and available in literature include time to crash (TTC) and its various modifications namely time exposed time to collision (TET) and time integrated time to collision (TIT), post-encroachment time (PET), delta-V (ΔV), relative speeds, or accelerations. These SSMs have not only been used for predicting the frequencies of crashes but also their severities (3, 4).

Research gap and contribution

Most of the SSMs that have been developed use the instantaneous value of the speed of the vehicle or acceleration (5) and then assume that to be the one after that instant to compute estimated values of time to collision. There are many SSMs built essentially around using constant velocity or acceleration to extrapolate the vehicle trajectory (6). This implies that the rotational motion of the vehicle is totally ignored. In fact, if we use the prevalent assumption and ignore the rotational kinematics or dynamics, we might produce false positives and false negatives.

Hence, it is very important to develop models that incorporate two-dimensional knowledge of the vehicle dynamics to be able to estimate the safety surrogate measures more accurately. Moreover, accidents are also related to road conditions, especially in inclement weather. Hence, the analysis should incorporate road conditions that can make traffic conflicts even more dangerous.

Heterogeneous traffic and traffic with weak lane discipline can render the dynamics of vehicle trajectories even more two dimensional (7–9).

This paper is written to address the two-dimensional nature of the traffic conflicts and shows how to incorporate that in analysis and modeling. In addition, the paper also uses road surface and tire interactions to assess prevalent adhesion coefficients so that those can be used for estimation as well. These two contributions render the safety surrogate measure-based analysis to be more thorough and comprehensive than with techniques currently available.

Problem statement

The problem statement that this paper is providing a solution to is as follows. Typically video data either taken by a drone or obtained by an infrastructure-supported camera are processed by performing image processing to obtain trajectories of various moving entities, such as vehicles and pedestrians. Given these trajectories, analysis needs to be performed to identify traffic conflicts as well as their severities. The conflict between a vehicle and a static object can be performed on each trajectory, and multiple vehicle trajectories at a time can be processed for their conflicts.

Road description

Figure 1 gives the visualization of the road surface in the mathematical structure of the problem. Parametrization for the two-dimensional surface in the ambient three-dimensional space is given by. The bi-directionality of the road is provided by an indicator function as

Combining the surface description with the direction indicating variable into a single vector function yields.

Trajectory description

Trajectories, as compared to road surfaces that are two-dimensional manifolds, are one-dimensional manifolds. Therefore, we can parametrize them using a single variable, taken as t here to represent time, when we represent a trajectory as. Once the set of all trajectories is known, the developed algorithm will identify traffic conflicts and ratings in terms of severity from the data.

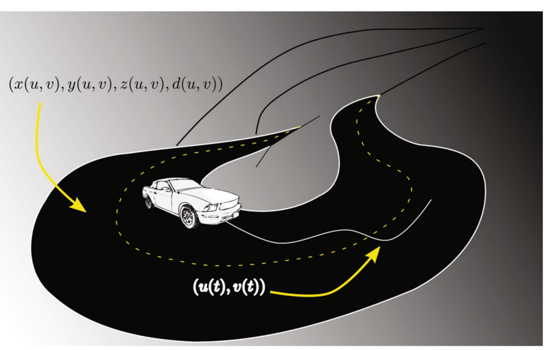

FIGURE 1 | Road surface and trajectories.

Kinematic analysis

The kinematic analysis involves a description of the motion of the vehicle with velocity inputs and outputs but without forces involved. The dynamic model with forces will contain adhesion coefficients and is covered in Section.

The Ordinary Differential Equations (ODE) model for the kinematic analysis is given by.

The kinematic model of the vehicle abstracted as a rolling disk or a wheel is shown in Figure 2. The vehicle can't slide perpendicular to the wheel in this model, and that develops the constraint equation that leads to the ODE model for the system.

This is a non-holonomic system, which means that the constraint on the motion is non-integrable. We have a constraint on the motion of the vehicle, as it can slide sideways, and that is why we have parallel parking maneuvers as being difficult. However, there is no constraint on where the vehicle can end up. For instance, a parallel parking maneuver in fact moves the vehicle from a given original state and renders the final state to be equivalent to a global lateral sliding motion but without contradicting that constraint. Hence, the constraint on the velocity is not integrable, as it does not lead to any constraints on the achievable states of the system.

(1)

Figure 2 (the middle and the right part) shows the derivation of the ODE model. The two inputs to the model are the linear velocity, v, and the angular velocity, ω, respectively. The linear velocity can only act in the direction the wheel is facing (the constraint). It is a three degrees of freedom model with two inputs, but the model is controllable (10).

Dynamic analysis

We will present the longitudinal dynamics first which will include wheel and vehicle models followed by combined longitudinal and lateral dynamics.

Longitudinal dynamics

We present the longitudinal wheel and vehicle dynamics emphasizing the role played by adhesion between the road and the tire.

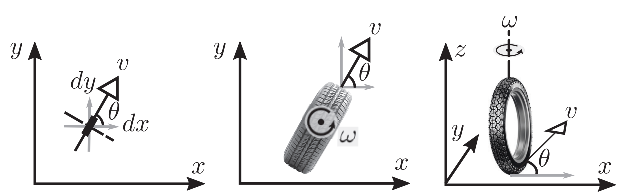

FIGURE 2 | Kinematic model.

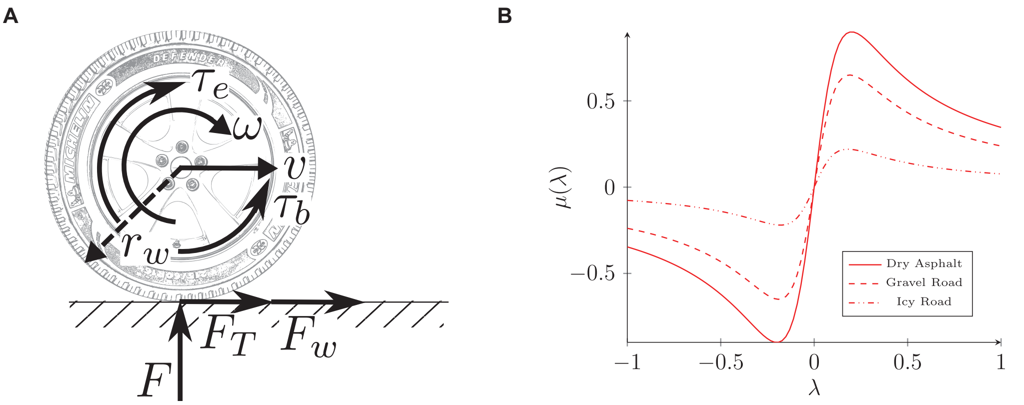

FIGURE 3 | Wheel dynamics and adhesion. **(A)** Wheel dynamics. **(B)** Adhesion coefficient wheel slip relationship.

Wheel dynamics

Vehicle dynamics depend on the traction between the tire and the ground. The tractive and the braking forces produced depend on the wheel slip that are defined as follows.
where v is the linear speed of the vehicle, ω the angular velocity of the tire, and r the radius of the tire, ω_v the linear velocity of the vehicle normalized by the tire radius. These variables are shown in **Figure 3A** that also shows the dynamics of the wheel. **Figure 3B** shows the relationship between the wheel slip and the adhesion coefficient. These variables and the additional variables in the figure are summarized in **Table 1**.

The adhesion coefficient which when multiplied by the normal reaction force gives the tractive force that drives the vehicle forward is a function of wheel slip defined by Equation 2. When vehicle, the wheel slip is positive, and when, the wheel slip is negative. The relationship between wheel slip and adhesion coefficient is shown in **Figure 3A**. The adhesion coefficient μ is positive for positive wheel slip λ which gives a forward tractive force (such as for acceleration), and alternately, μ is negative for negative wheel slip λ which gives a backward tractive force (such as for deceleration), when we apply brakes which causes ω to become less thanω .

TABLE 1 | Notation for the free body diagram of a parked car.

Symbol	Meaning
rw	Wheel radius
ω	Wheel angular speed
v	Vehicle Linear Speed
ω*v*	Normalized Vehicle Speed
τ*e*	Torque from engine on the wheel
τ*b*	Braking torque
FT	Tractive force
Fw	Wheel viscous friction
F	Normal ground reaction force
μ	Adhesion coefficient
λ	Wheel slip
Jw	Wheel moment of inertia

There are various approximate formulas that relate μ to λ (11, 12). Burckhardt's and Pacejka's models give a formula for the relationship. However, it has too many parameters. We choose a simpler model for our purpose as (13), where μ_0 and λ_0 are the peak adhesion coefficient of the curve and its corresponding λ value, respectively.

For different road conditions, we get different sets of the parameters. Some curves are shown in **Figure 3A** for various road conditions that were obtained by using the parameters related to those road conditions (11, 14). We have for tractive force as

The rotational dynamics at the wheel are:
where we have used the over dot convention to show derivative of a variable which is a function of single variable time, with respect to time, and

Vehicle dynamics

The vehicle dynamics are illustrated in **Figure 4A**. The vehicle body coordinate system is shown by, whereas the world coordinate is. The yaw angle of the vehicle is given byθ, which is the angle between the *X* world axis and *x* vehicle axis. The vehicle velocity component in the direction of x is v_x, while the component in the direction of y is v_y. The velocity vector is in the direction-making angleα , called the slip angle, with respect to the *x* axis.

Figure 4B shows the forces and torque on the vehicle. Force F_x is the longitudinal tractive force from the wheel. Force F_y is the lateral tractive force from the wheel. The tractive forces are dependent on wheel slip. However, they also depend on the slip angleα . If F is the normal force, we have.

Figures 5A, B shows the plots of and for a typical road condition.

For different road conditions, how the plots change is shown in **Figure 6**.

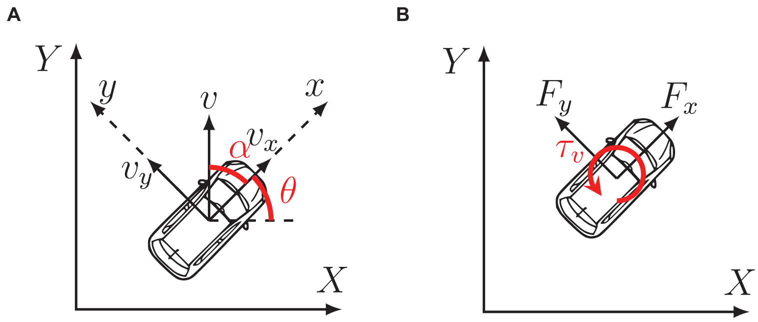

FIGURE 4 | Vehicle dynamics. **(A)** Vehicle dynamics with adhesion. **(B)** Vehicle force and torque.

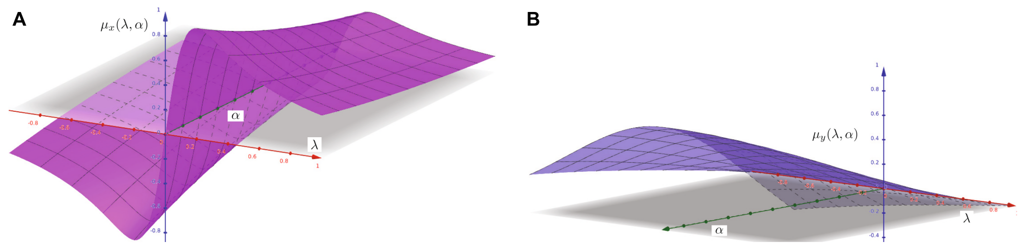

FIGURE 5 | Longitudinal and lateral adhesion coefficients. **(A)** $\mu_x(\lambda, \alpha)$. **(B)** $\mu_y(\lambda, \alpha)$.

Although the actual formula for the two coefficients is quite complicated (12), we can use the following simplified formula.

Combined longitudinal and lateral dynamics

We now present the combined longitudinal and lateral dynamics where we study the wheel and vehicle dynamics continuing to emphasize the role played by adhesion between the road and the tire.

We have a fixed road reference inertial frame and a frame that moves with the vehicle. Newton's laws are followed in an inertial frame, and hence. Mass m is an invariant in these two frames. We can transform this equation to the vehicle moving frame and can obtain (15, 16).

In our model, we have, where an overhead dot implies differentiation with respect to time, and double overhead dot implies the second derivative with respect to time. In component form, our model becomes

We can apply Newton's laws to obtain the dynamics. Using the expressions for accelerations in the x and y directions from Equation 6, and then applying Newton's laws in those two directions taking m as the vehicle mass, we obtain

Taking the moments about the center of gravity (e.g.) and denoting I_z as the moment of inertia of the vehicle about the c.g in the z direction, we apply rotational Newton's law to obtain.

The vehicle speed in terms of road/world coordinates is obtained by a rotation of angle θ of the representation in the vehicle frame.

SSM analysis

We will perform the analysis and compare different techniques using the time to collision (TTC) measure.

Standard analysis

First-order model

Using standard analysis, at any given time t, TTC is measured by assuming the current value of the velocity to remain till collision, and then the time to collision is calculated based on that assumption.

Figure 7A shows the dynamics we encounter for this standard analysis. At any given time, the vehicle under

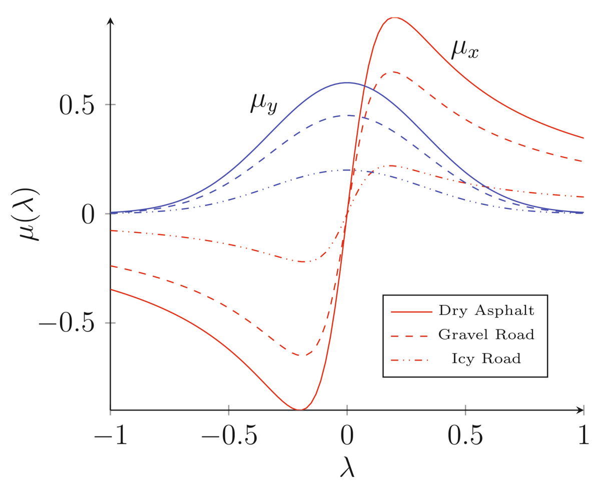

FIGURE 6 | Different road conditions.

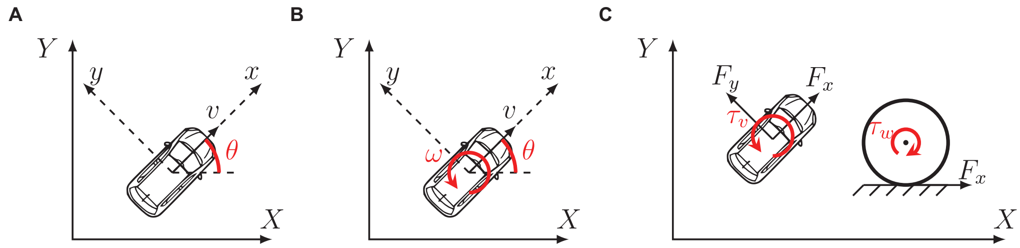

FIGURE 7 | TTC setup. **(A)** Standard. **(B)** Kinematic. **(C)** Dynamic.

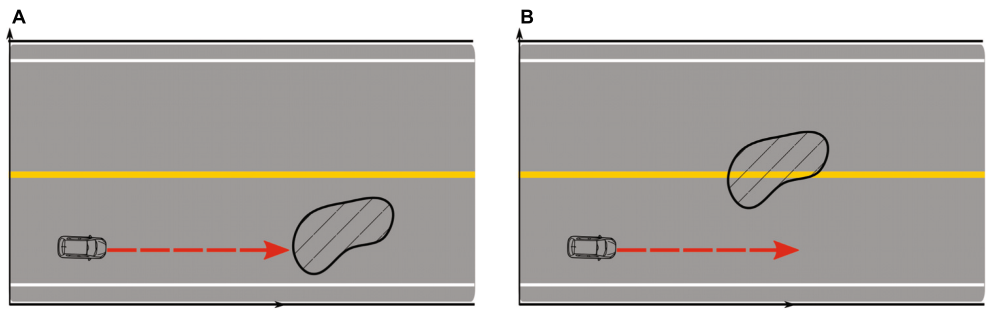

FIGURE 8 | Standard model for vehicle and an obstacle. **(A)** Collision case. **(B)** Non-collision case.

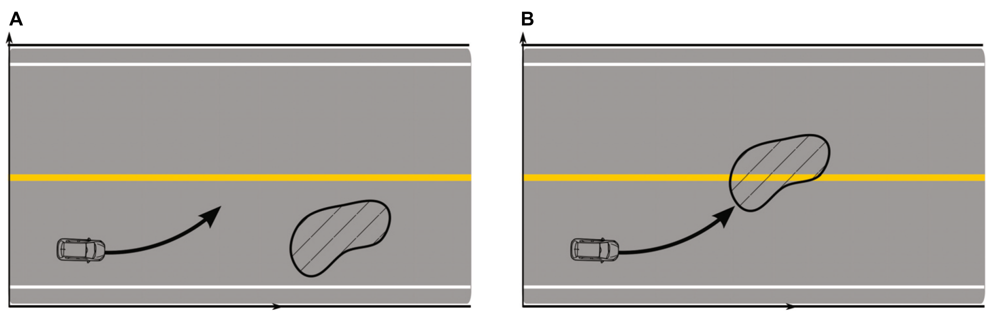

FIGURE 9 | Kinematic model for vehicle and an obstacle. **(A)** Collision case. **(B)** Non-collision case.

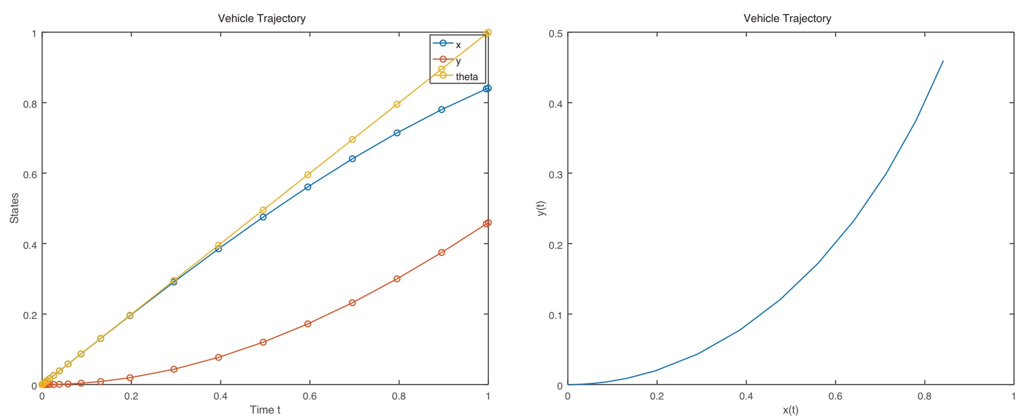

FIGURE 10 | Vehicle trajectory plots with angular rate added.

study would be at coordinate with a yaw angle of θ. In standard analysis, we use the current velocity as a constant for estimating TTC. Hence, the dynamics for the trajectory are, and, where the angle θ is a constant which is its initial value. Hence, the trajectory is a straight line.

Second-order model

A second-order model can also be used (5), where the initial acceleration is kept constant. In that model, add one more differential equation as, and keep *a* constant. In this case, *v* will vary, but the motion will still be in a straight line.

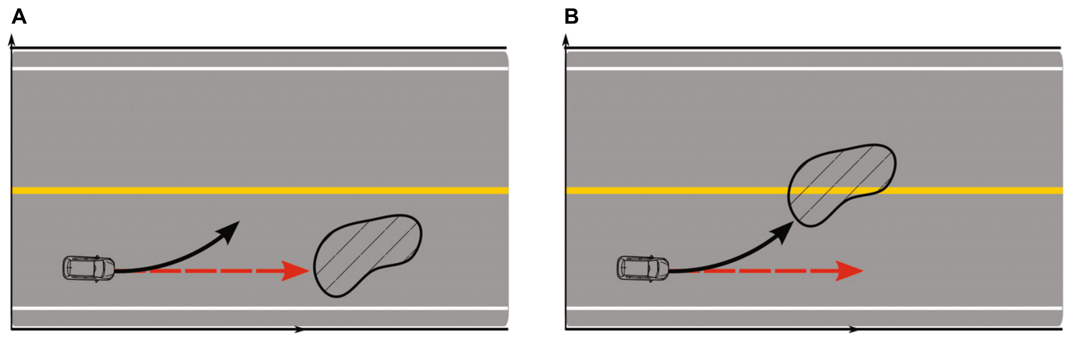

FIGURE 11 | Kinematic model for vehicle and an obstacle. **(A)** False positive. **(B)** False negative.

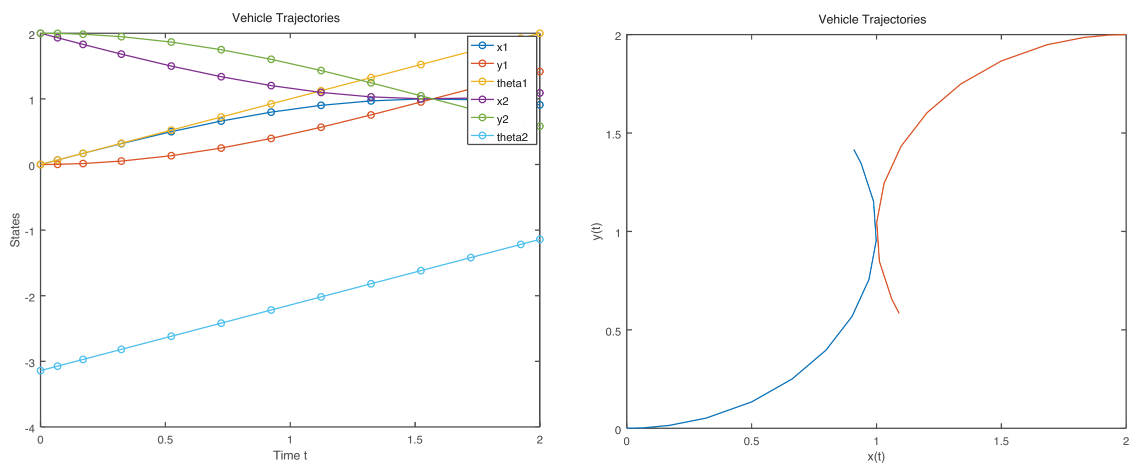

FIGURE 12 | Vehicle trajectory plots for interacting vehicles.

Kinematic analysis

First-order model

The differential equation model for the kinematics is given by, and. For the TTC analysis using this model, we keep the initial linear speed and angular velocity constant, as compared to only the linear velocity as in the case of standard analysis.

Figure 7B shows the dynamics we encounter for this kinematic analysis. At any given time, the vehicle under study would be at coordinate with a yaw angle of θ. In kinematic analysis, we use the current linear speed and angular velocity as constants for estimating TTC. Hence, the dynamics for the trajectory are given in Equation 9. The trajectory is a circular arc satisfying, where v is the constant initial speed and ω, is the constant initial angular speed.

Second-order model

A second-order model can also be used, where the initial linear acceleration is kept constant as well as the initial angular acceleration. In that model, we modify Equation 9

by adding two more differential equations as, and keep a constant, as well as and keep α constant. In this case, v and ω both will vary.

Dynamic analysis

Dynamic analysis involves inherently a second-order system as forces are used in the model. We use the differential equations from Section for this modeling and take constant values of τ_w and τ_v for the analysis which would be their initial values at time t. Figure 7C shows the dynamic setup for TTC.

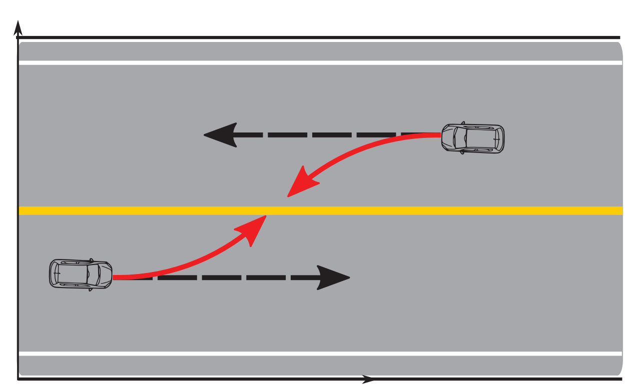

FIGURE 13 | Vehicle trajectories: false negative.

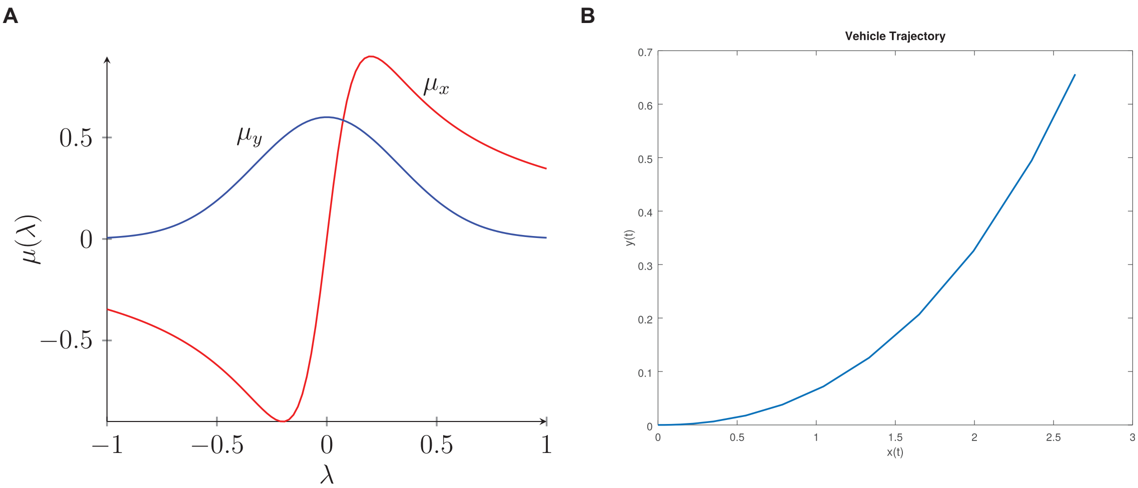

FIGURE 14 | Dry asphalt. **(A)** Road surface condition. **(B)** Trajectory plot.

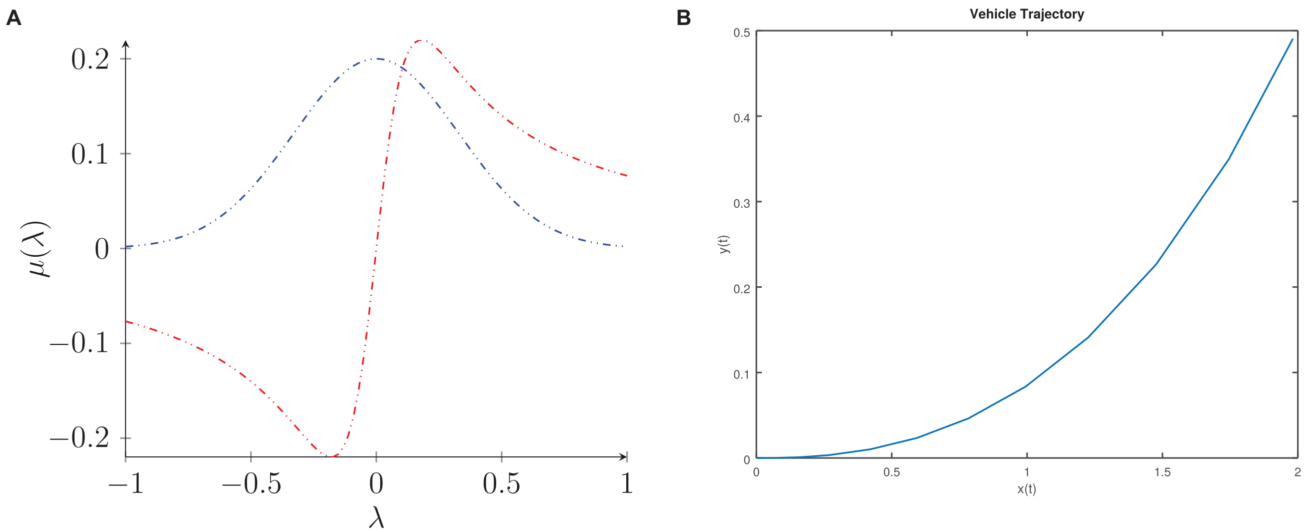

FIGURE 15 | Icy road. **(A)** Road surface condition. **(B)** Trajectory plot.

Numerical examples and results

In the numerical case study example, we will analyze a vehicle object interaction, as well as between two vehicles.

Vehicle obstacle example

We will study the placement of the obstacle at two different locations to see the difference in the results when we use different models for the analysis of TTC.

Standard analysis

Using the standard model, we use the constant speed of the vehicle equal to the current speed at a given time *t*.

With the same initial conditions for the two scenarios for the vehicle but different obstacle locations we see a collision in one case and non-collision in the other, as shown in **Figures 8A, B**. Hence, the TTC can be calculated for the collision case.

Kinematic analysis

Using the kinematic model, we use constant linear speed and angular speed of the vehicle equal to the current values at a given time *t*.

With the same initial conditions for the two scenarios for the vehicle but different obstacle locations, we see a collision in one case and non-collision in the other for the kinematic analysis case also. Hence, the TTC can be calculated for the collision case like before but for the different obstacle locations, as shown in **Figures 9A, B**.

The simulation plots for the trajectory are shown in **Figure 10**.

Comparison of standard and kinematic analysis

When we compare the standard and the kinematic case, we realize that if the initial condition in fact has an angular speed, then the standard model produces false positives and negatives, as shown in **Figures 11A, B**.

Vehicle vehicle example

This subsection will illustrate how we can use the kinematic model for performing collision analysis from a traffic conflict involving a vehicle and another vehicle. The vehicle-to-vehicle interaction is similar to the vehicle with obstacle case, in the sense that we use the same non-holonomic ODEs for both vehicles starting from their respective initial conditions, and then analyze the trajectories for collisions, or if there is no collision then a closeness value.

To illustrate this method, we will demonstrate the case of a true negative where longitudinal only analysis would show no collision where in fact a collision takes place.

We run the simulation and obtain the false negative as shown in **Figure 12** and recreated in **Figure 13**.

Dynamic analysis

In this section, we will show that when we include read surface conditions, the vehicle trajectories change. Hence, we should use road surface conditions in extrapolating vehicle trajectories for computing SSMs such as the TTC.

We performed a simulation for two different road conditions to see the change in the trajectories. We started with the same initial conditions as in standard and kinematic analyses.

Figure 14A shows the road surface conditions for dry asphalt for the simulation where the vehicle trajectory is shown in **Figure 14B**, whereas **Figure 15A** shows the road surface conditions for dry asphalt for the simulation where the vehicle trajectory is shown in **Figure 15B**. We can clearly see the change in trajectory when the road condition is changed for the same initial conditions.

Conclusion

In this paper, we have shown analysis for safety surrogate measures that involve more detailed vehicle modeling levels, where one model is based on a non-holonomic constrained system and the other model is based on including road surface and tire interactions. The case study examples show that the standard system can have false positives and false negatives and hence these new models are important in creating more accurate safety surrogate measures which then can lead to better estimates of collisions.

Author contributions

This manuscript was written and the algorithm was designed by the first author, whereas the problem statement and the road surface condition use for SSMs were jointly developed by both authors.

Acknowledgments

The first author is thankful to IIT Delhi for the visiting professor position as well as to Dr. Geetam Tiwari for the research opportunity and collaboration.

References

1. Castillo E. *Extreme value theory in engineering*. Amsterdam: Elsevier (2012).
2. De Haan L, Ferreira A, Ferreira A. *Extreme value theory: an introduction*. Berlin: Springer (2006). 21 p.
3. Arun A, Haque MM, Bhaskar A, Washington S, Sayed T. A systematic mapping review of surrogate safety assessment using traffic conflict techniques. *Accid Anal Prev.* (2021) 153:106016.
4. Tarko A. *Measuring road safety with surrogate events*. Amsterdam: Elsevier (2019).
5. Ozbay K, Yang H, Bartin B, Mudigonda S. Derivation and validation of new simulation-based surrogate safety measure. *Transp Res Record.* (2008) 2083:105–13.
6. Nadimi N, Ragland DR, Mohammadian Amiri A. An evaluation of time-to-collision as a surrogate safety measure and a proposal of a new method for its application in safety analysis. *Transp Lett.* (2020) 12:491–500.
7. Budhkar AK, Maurya AK. Characteristics of lateral vehicular interactions in heterogeneous traffic with weak lane discipline. *J Mod Transp.* (2017) 25:74–89.
8. Gaddam HK, Rao KR. A two-sided lateral gap continuum model and its numerical simulation for non-lane based heterogeneous traffic environment. *J Intell Transp Syst.* (2020) 24:635–53.
9. Mallikarjuna C, Rao KR. Heterogeneous traffic flow modelling: a complete methodology. *Transportmetrica.* (2011) 7:321–45.
10. Slotine J-JE, Li W. *Applied nonlinear control*. Englewood Cliffs, NJ: Prentice Hall (1991).
11. Kiencke U, Nielsen L. *Vehicle modelling*. Berlin: Springer (2005).
12. Pacejka H. *Tire and vehicle dynamics*. Amsterdam: Elsevier (2005).
13. Lin J-S, Ting W-E. Nonlinear control design of anti- lock braking systems with assistance of active suspension. *IET Control Theory Appl.* (2007) 1:343–8.
14. Kachroo P, Tomizuka M. Vehicle traction control and its applications. *California PATH Research Paper, UCB-ITS-PRR-94- 08(ISSN 1055-1425)*. Oakland, CA: California Digital Library (1994). p. 3–44.
15. Goldstein H, Poole C, Safko J. *Classical mechanics*. Reading, MA: Wesley (1980). 426 p.
16. Jazar RN. *Vehicle dynamics*. Berlin: Springer (2008). 1 p.