

CASE STUDY

Application of genetic algorithm (GA) and design of experiment (DOE) to solve the inventory dynamic lot-sizing problem: a case study

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In inventory planning, the Dynamic Lot-sizing (DLS) problem is a challenging problem characterized by time-varying demand and quantity-varying costs. Optimization methods using mathematical models have difficulty solving this problem due to its combinatorial nature and dynamic characteristics. Genetic algorithm (GA) is a meta-heuristic algorithm that can search for near-optimal solutions in a large solution space, thereby effectively solving this problem. In this study, the mathematical model of the DLS problem has been constructed. Based on the model, a GA algorithm is developed and used to solve the DLS problem with the objective to minimize the total inventory cost. The parameters of the GA algorithm are optimized by using Design of Experiment (DOE). The results show that GA is better than the currently used heuristic algorithm, and GA with DOE is better than GA.

Keywords: dynamic lot-sizing (DLS), multi-item multi-period inventory, all-unit discount, genetic algorithm, design of experiment

Introduction

Effective inventory management is crucial for the smooth operation of businesses, as it ensures optimal stock levels while minimizing costs. In the context of an equipment processing company in Vietnam, efficient inventory control holds significant importance. Currently, the company relies on an experiential inventory model for ordering, which lacks consideration for demand forecasting and cost-related factors.

However, the study has recognized some problems with this model, which are: the inappropriate planning model leads to the situation of excessive inventory, which increases inventory costs; the cost of raw materials is increasing,

and continuing with the current lot sizing planning of the company will result in higher inventory costs and a shortfall in purchasing budget; the warehouse space for materials is limited, and ineffective inventory control can lead to warehouse overload.

A model is proposed to address this problem. An inventory system is developed for multiple materials with discrete, known demand, with storage, budget, and order quantity constraints, with an All Unit Discount (AUD) for each item.

The model determines the optimal order quantity of products to minimize the total inventory cost during the planning period. Real-life situations increase the complexity of the model; the Genetic Algorithm (GA) algorithm is used to solve the proposed mathematical model.

The GA is an optimization method inspired by natural selection, capable of finding optimal or near-optimal solutions by evolving a population of potential solutions through selection, crossover, and mutation. It is widely used for solving complex problems in various fields.

Design of experiments (DOEs) is used to optimize the parameters of the GA model. The results of the heuristic methods being used, of the GA, and of the GA with DOE are compared to evaluate their effectiveness.

Literature review & research methodology

Literature review

Inventory dynamic lot-sizing

The stochastic inventory models first appeared in the literature in the early 1950s in specialized papers by Arrow et al. (1) and Dworetzky et al. (2, 3). These models were developed to capture the uncertainty and variability that exist in most real-world inventory situations but are often assumed away in deterministic inventory models. Due to the combination of uncertainty in demand and/or lead time, the fundamental mathematical structure for stochastic inventory models is more complex than that for deterministic models and requires management to have policy statements regarding system inventory performance measures, such as service level or fill rate.

Inventory models can be classified into two main categories by Aggarwal (4): static and dynamic models. In a static model, the parameters remain constant over time, such as constant demand rate, constant lead time, and fixed costs. On the other hand, dynamic models involve parameters that continuously change over time, such as varying demand rates, delivery lead times, and cost factors that affect total inventory costs. Dynamic models typically require more complex and challenging calculations compared to static models.

Lot sizing models can be classified into static and dynamic models. Static models, such as the Economic Order Quantity (EOQ) model, plan lot sizes with constant demand and inventory costs. Dynamic models plan lot sizes with variable demand and inventory costs.

Genetic algorithm

Genetic Algorithm (GA) is a meta-heuristic algorithm that starts with a population of potential solutions represented as chromosomes. Through selection, crossover, and mutation operators, the algorithm evolves the population over many generations. This process continues until a stopping condition is reached. GAs are effective in exploring large search spaces and finding near-optimal solutions for complex

problems. GAs are used in a wide variety of fields and are known for their adaptability and flexibility.

Notably, GA has been utilized in previous research studies to address specific inventory-related challenges. For instance, Mondal et al. (5) developed a multi-item fuzzy EOQ model, and Maiti et al. (6) investigated the optimization of imperfect production processes for damageable items. Both studies successfully employed GA to solve their respective mathematical models. The versatility and effectiveness of GA make it a valuable tool in addressing complex inventory management problems and optimizing decision-making processes.

Research methodology

The research methodology includes the following steps.

1. Developing the mathematical model of the dynamic lot-sizing (DLS) problem.
2. Developing the GA algorithm to solve the DLS problem.
3. Applying the GA algorithm to solve the DLS problem.
4. Using DOE to optimize the parameters of the GA model.
5. Applying the GA algorithm with optimized parameters to solve the DLS problem.

Mathematical model of the dynamic lot-sizing problem

Consider a company that plans inventories for several materials with known demand that varies in cycles over a finite planning horizon of N periods. The initial inventory level for all items is zero, and only one order is placed for a particular item in each cycle.

Additionally, there are limitations on the order quantity of each item, and ordered quantities are delivered in batch sizes without the possibility of splitting batches. The company also has restrictions on shortages, storage space, and the available budget. Quantity discount policies are implemented based on the prevailing circumstances. The objective is to determine the optimal order quantities for the products, variable demand in each period, to minimize the total cost of the inventory system while satisfying the given constraints. The problem to be solved is a DL problem with three items, $i = 1 \div 3$, with 12 periods, $j = 1 \div 12$.

Assumptions

1. The initial inventory level for the first cycle of all items is 0.

2. The price level is known and can vary depending on the order quantity.
3. There are fixed and unchanged unit costs, and there is no price inflation.

Parameters

Item index i : $i = 1, 2, 3$

Period index j : $j = 1 \div 12$

Breakpoint index k : $k = 1, 2, 3$

TC: total inventory cost (USD)

TO: total purchasing cost (USD)

TH: total holding cost (USD)

TP: total ordering cost (USD)

O_i : ordering cost per order time of item i in period j (USD/one order)

X_{ij} : initial inventory level of item i in period j (in $j = 1$ the initial inventory of all items is zero)

D_{ij} : demand of item i at period j (unit)

T_j : total time elapsed up to period j

I_i : inventory level of item i at time t ($T_j < t < T_{j+1}$)

H_i : inventory holding cost per unit of item i during a period (USD/unit/year)

$q_{i,k}$: the k th discount breakpoint of i th product ($q_{i,1} = 0$)

$P_{i,k}$: The purchase price of item i at breakpoint k under the AUD policy

S : total available storage space (m^2)

s_i : required warehouse space per unit of i th product (m^2 /unit)

C : total available budget (\$)

M_1 : an upper bound for Q_{ij} .

Decision variables

Q_{ij} : ordering quantity of item i in period j .

W_{ij} : a binary decision variable; determining whether there is an order placement in a period; set equal one if a purchase of an item i is made in period j , and zero otherwise.

$U_{ij,k}$: a binary decision variable; determine if there is an order placement at price point k ; set equal one if item i is purchased at price breakpoint k in period j , and zero otherwise.

Objective function

The beginning inventory of item i :

$$X_{ij+1} = X_{ij} + Q_{ij} - D_{ij} \quad (1)$$

The total ordering cost:

$$TO = \sum_{i=1}^m \sum_{j=1}^{N-1} O_i W_{ij} \quad (2)$$

In order to obtain the total holding cost, we first calculate $I_i(t)$ for ($T_j < t < T_{j+1}$):

$$I_i(t) = (X_{i,j} + Q_{i,j}) - \frac{D_{i,j}(t - T_j)}{T_{j+1} - T_j} \quad (3)$$

Therefore, the holding cost of item i in the interval $[T_j, T_{j+1}]$ is

$$TH = H_i \int_{T_j}^{T_{j+1}} I_i(t) dt \quad (4)$$

Inserting Equations (3) in (4):

$$TH = \sum_{i=1}^m \sum_{j=1}^N H_i (T_{j+1} - T_j) [X_{ij} Q_{ij} + \frac{D_{ij} T_j}{T_{j+1} - T_j} - \frac{D_{ij}(T_{j+1} + T_j)}{2(T_{j+1} - T_j)}] \quad (5)$$

The purchasing cost corresponded to AUD policies obtained as follows:

$$\begin{cases} P_{i,1}; 0 < Q_{ij} \leq q_{i,2} \\ P_{i,2}; q_{i,2} < Q_{ij} \leq q_{i,3} \\ \vdots \\ P_{i,K}; q_{i,K} < Q_{ij} \end{cases}$$

The purchasing cost under the AUD scheme:

$$TP = \sum_{i=1}^m \sum_{j=1}^{N-1} \sum_{k=1}^K P_{i,k} Q_{ij} U_{ij,k} \quad (6)$$

The total cost of the inventory control problem includes total ordering cost, total holding cost, and total purchasing cost:

$$TC = TO + TH + TP \quad (7)$$

From Equations (2), (5), (6), and (7), the objective function is as follows.

$$\begin{aligned} \text{Min } TC = & \sum_{i=1}^m \sum_{j=1}^{N-1} O_i W_{ij} + \sum_{i=1}^m \sum_{j=1}^N H_i (T_{j+1} - T_j) \\ & [X_{ij} + Q_{ij} + \frac{D_{ij} T_j}{T_{j+1} - T_j} - \frac{D_{ij}(T_{j+1} + T_j)}{2(T_{j+1} - T_j)}] \\ & + \sum_{i=1}^m \sum_{j=1}^{N-1} \sum_{k=1}^K P_{i,k} Q_{ij} U_{ij,k} \end{aligned}$$

Constraints

$$X_{i,1} = 0, \quad i = 1 \div M$$

$$\sum_{i=1}^m (X_{ij} + Q_{ij})s_i \leq S; j = 1 \div N$$

$$\sum_{i=1}^m Q_{ij}P_i \leq C; j = 1 \div N$$

$$Q_{i,j} \geq M_i; i = 1 \div m \text{ \& } j = 1 \div N$$

$$\sum_{k=1}^K U_{i,j,k} = 1; i = 1, \div m \text{ \& } j = 1 \div N$$

$$X_{ij} \geq 0; i = 1 \div m \text{ \& } j = 1 \div N$$

$$Q_{ij} \geq 0 \forall i, j$$

$$W_{ij} \in \{0, 1\} \forall i, j$$

$$U_{i,j,k} \in \{0, 1\} \forall i, j, k$$

$$B_i V_i \in N^*$$

Constraint 1 presents the initial inventory of all items is zero. Constraint 2 is related the area of the storage that was enough to store the ordered items. Constraint 3 is the allowed budget for the order. Constraint 4 presents an upper bound for $Q_{i,j}$. Constraint 5 represents that there must be an order. Constraint 6 ensures the positive inventory levels. The remaining constraints help tighten the model.

The company is currently planning based on experience. The current objective value is as follows.

$$TC = 25, 182.$$

The GA model for solving the dynamic lot-sizing problem

The above DL problem is a non-linear problem with a solution space of $11^{12 \times 3}$, or 11^{36} . The GA model is used to solve the problem.

The method of coding

Within this study, the chromosomes employed are represented as matrix, denoting the order quantities of items ($r_{i,j}$) during each respective period. To provide a visual illustration, [Figure 1](#) portrays the chromosome structure for a scenario comprising four products and four periods. In this depiction, the rows and columns correspond to the number of items and periods, respectively.

$$Q = \begin{bmatrix} r_{1,1} & r_{1,2} & \dots & r_{1,12} \\ r_{2,1} & r_{2,2} & \dots & r_{2,12} \\ r_{3,1} & r_{3,2} & \dots & r_{3,12} \end{bmatrix}$$

FIGURE 1 | The structure of a chromosome.

The GA procedure

The GA parameters are shown in [Table 1](#) as follows.

The GA factors are shown in [Table 2](#) as follows.

The GA pseudo code is as follows.

Begin

Set parameters (P, P_s, P_c, P_m, P_e, n)

Randomly initialize population P

OF = TC(P)

Best₀ = min OF

Fit = 1/OF²

$g = 0$;

While ($g < n$)

TABLE 1 | The Genetic Algorithm (GA) parameters.

Parameters	Meaning
P	Population size
P_s	Selection rate
P_c	Crossover probability
P_m	Mutation probability
P_e	Elitism rate
n	Max number of iterations <i>with unchanged OF</i>

TABLE 2 | The GA factors.

Factors	Meaning
TC	Total cost
OF	Objective function
Best	The best in population
Fit	Fitness function
EP	Elite population
PE	Number of individuals selected to EP
SM	Selection method to generate EP
CP	Crossover population
CM	Crossover method
MP	Mutation population
MM	Mutation method
GP	Generated population after crossover & mutation
CRP	The current population
NP	The next population
RM	Replacement method to make NP
g	Iteration counter <i>with unchanged OF</i>

Select individuals from CRP to EP (Using SM with selection probability P_s)

Crossover (Using CM, chromosomes in EP chosen to CP if random $[0,1] < P_c$)

Mutation (Using MM, chromosomes in EP chosen to MP if random $[0,1] < P_m$)

Calculate the objective function for new individuals GP;

Select individuals from GP to NP (Using RM with elitism probability P_e)

Update values (CRT; Best);

If $\text{Best}_{i+1} - \text{Best}_i \geq 0$, **Then** $g = g + 1$, **Else** $g = 0$

EndWhile

Return CRP; Best

End

The steps involved in the proposed GA algorithm are as follows.

Step 1. Establish the GA parameters.

Step 2. Initialize the population randomly, consisting of P individuals.

Step 3. Evaluate the objective function to quantify the fitness of each individual within the population.

Step 4. Select individuals for the EP from CRP by using SM with selection probability P_s .

Step 5. Apply the crossover operator to each pair of chromosomes, guided by the CM & P_c .

Step 6. Implement the mutation operation on each chromosome, guided by the MM & P_m .

Step 7. Generate the GP, including offspring generated through crossover and mutation.

Step 8. Generate the next population NP by replacing bad chromosomes in CRP with chromosomes in GP by using RM with elitism probability P_e .

Step 9. Check if the stopping criterion has been met. If the criterion is satisfied, terminate the algorithm. If not, proceed to Step 3 to continue the iterative process.

Applying the GA model for solving the DLS problem

The GA parameters & factors

In order to solve the DLS problem, the GA parameters are chosen in [Table 3](#) as follows.

The GA factors are chosen in [Table 4](#) as follows.

The GA operators

The selection operator

The selection operator selects chromosomes from the CRP to EP by using the *Roulette Wheel Selection* method with selection probability P_s as presented in reference (7).

TABLE 3 | The GA parameters and factors.

Parameters	Values
P	30
P_s	0.8
P_c	0.8
P_m	0.2
P_e	0.2
n	100

TABLE 4 | The GA factors.

Factors	Meaning
SM	Roulette wheel selection
CM	Uniform
MM	Two-point swapping
RM	Elitism selection

The crossover operator

Pasandideh et al. (8) also proposed a commonly used crossover operator called the uniform crossover. To perform the crossover operation, firstly, a binary chromosome is randomly generated. Then, within the generated binary chromosome, the genes with a value of one are considered. The genes of the parents are crossed over in such a way that they have the same positions with a value of one. An example is shown in [Figure 2](#).

The mutation operator

The two-point swapping mutation operator, as mentioned by Tsai in (9), is applied by randomly selecting two points on the chromosome and exchanging their positions with each other. An example is shown in [Figure 3](#).

The replacing operator

Using the elitism selection method, the top $P_e \times P$ best individuals are preserved, which in this case would be $0.2 \times 30 = 5$ best individuals from the previous population, without undergoing crossover and mutation. Then, the remaining 25 chromosomes are selected for the next generation using the roulette wheel selection method.

The stopping criterion

After n consecutive iterations without any improvement, the model end. And the final result of TC is shown in the graph below in [Figure 4](#).

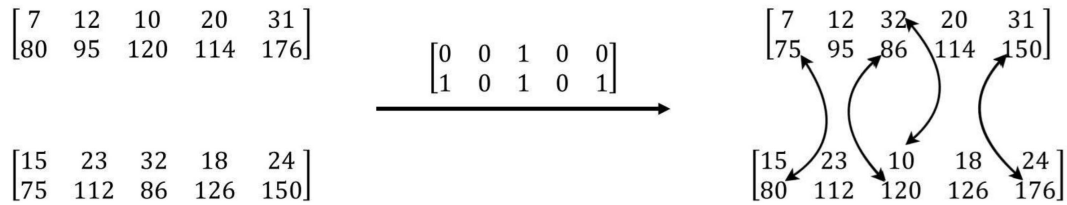


FIGURE 2 | An example of the crossover operator.

$$Q_{26} = \begin{bmatrix} 2 & 3 & 7 & 10 & 9 & 2 & 5 & 2 & 8 & 3 & 6 & 9 \\ 2 & 10 & 8 & 7 & 8 & 0 & 10 & 3 & 4 & 3 & 9 & 10 \\ 8 & 0 & 5 & 3 & 8 & 2 & 8 & 1 & 6 & 3 & 9 & 10 \end{bmatrix}$$

↓

$$Q'_{26} = \begin{bmatrix} 2 & 3 & 7 & 10 & 0 & 2 & 5 & 2 & 8 & 3 & 6 & 9 \\ 2 & 10 & 8 & 7 & 8 & 0 & 10 & 3 & 4 & 3 & 9 & 10 \\ 8 & 9 & 5 & 3 & 8 & 2 & 8 & 1 & 6 & 3 & 9 & 10 \end{bmatrix}$$

FIGURE 3 | An example of the mutation operation.

The algorithm's results regarding the optimal batch size and the objective function of inventory cost are as follows.

$$Q = \begin{bmatrix} 7 & 2 & 0 & 0 & 9 & 0 & 0 & 1 & 2 & 5 & 0 & 2 \\ 7 & 3 & 4 & 0 & 3 & 6 & 7 & 9 & 9 & 0 & 1 & 7 \\ 4 & 7 & 6 & 0 & 6 & 0 & 4 & 0 & 1 & 4 & 0 & 4 \end{bmatrix}$$

$$TC = 18,336.725$$

Applying the GA with DOE to solve the DLS problem

The parameters of the above GA model were selected empirically, resulting in suboptimal results. To

optimize the model parameters, an experimental design approach is employed.

The 2^k experiment

Initially, a 2^k experiment is conducted with the input factors P , n , P_s , P_c , P_m , and P_e , and the output factor TC, representing the total inventory cost. The levels of the input factors are chosen as follows in Table 5.

With the sample size of 3, the total number of experiments is 36. After the experiment, the ANOVA table is shown in Figure 5.

TABLE 5 | Two levels of the input factors.

Factor	Value	
	Low	High
P	30	50
P_s	0.33	0.8
P_c	0.7	0.9
P_m	0.15	0.25
P_e	0.15	0.3
n	50	100

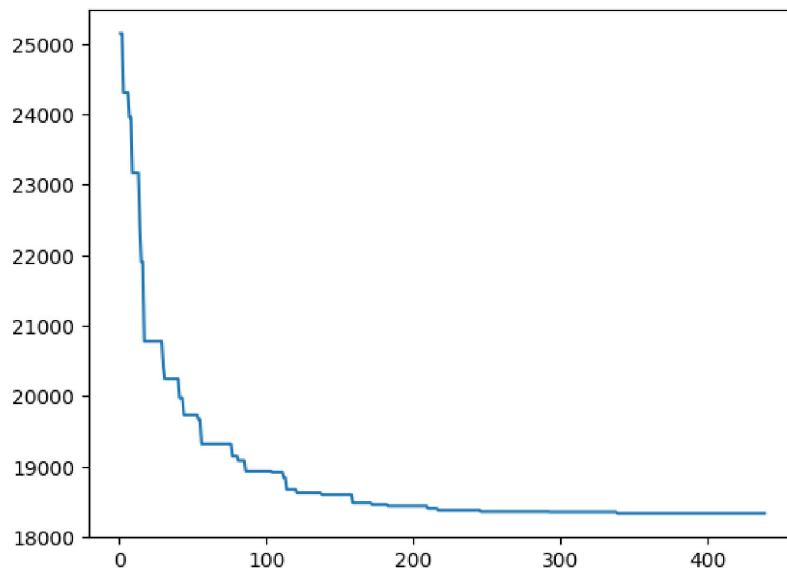


FIGURE 4 | The result of the objective function of the GA model.

Analysis of Variance

Source	DF	Contribution	Adj SS	Adj MS	F-Value	P-Value
Model	6	47.80%	2985639	497607	4.43	0.003
Linear	6	47.80%	2985639	497607	4.43	0.003
P	1	12.52%	781751	781751	6.95	0.013
n	1	0.00%	230	230	0.00	0.964
Pc	1	3.74%	233450	233450	2.08	0.160
Pm	1	0.49%	30824	30824	0.27	0.605
Ps	1	21.03%	1313546	1313546	11.69	0.002
Pe	1	10.02%	625839	625839	5.57	0.025
Error	29	52.20%	3259867	112409		
Lack-of-Fit	5	9.75%	608790	121758	1.10	0.385
Pure Error	24	42.45%	2651077	110462		
Total	35	100.00%				

FIGURE 5 | Analysis of variance of the experiment.

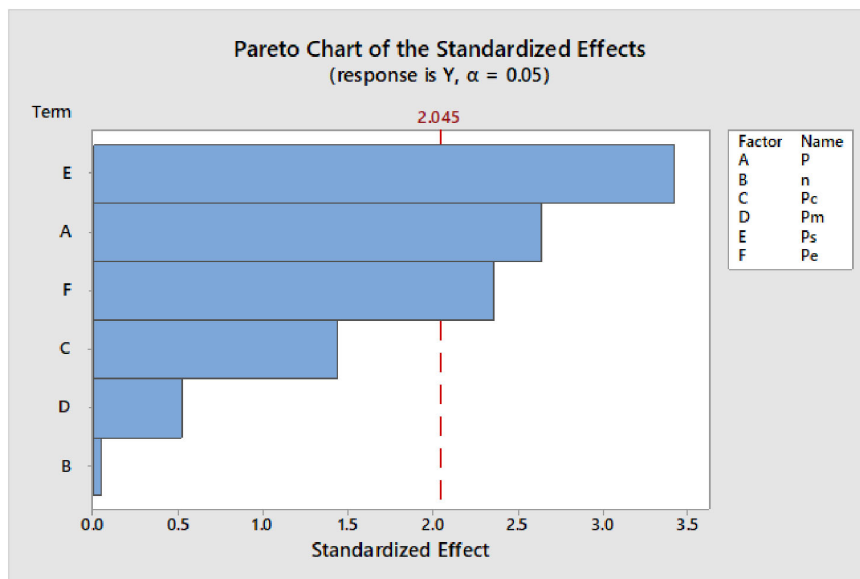


FIGURE 6 | The Pareto chart illustrates the level of influence of factors from high to low.

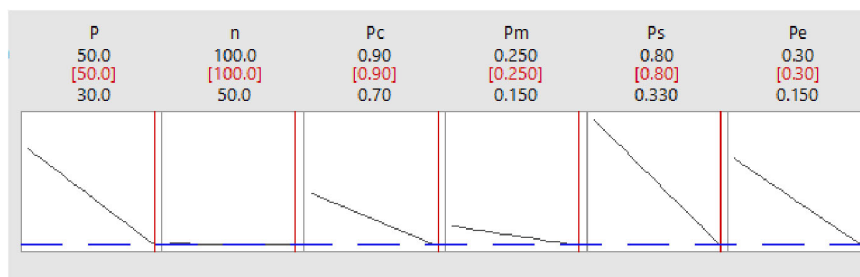


FIGURE 7 | The values of parameters to give the minimum objective value.

TABLE 6 | Five levels of the input factors.

Factor		P	P_s	P_e
Levels	1	30	0.33	0.1
	2	40	0.45	0.15
	3	50	0.5	0.2
	4	60	0.8	0.25
	5	70	0.9	0.3

From the above ANOVA table, with α of 0.05, the affected factors are factors P , P_s , and P_e . The following **Figure 6** Pareto chart illustrates the level of influence of factors from high to low.

The optimized values of parameters are shown in **Figure 7**.

$$P = 50; n = 100; P_c = 0.9; P_m = 0.25; P_s = 0.8; P_e = 0.3$$

The multivariate experiment

A multivariate experiment is conducted with the three affected factors P , P_s , and P_e . The levels of the input factors are chosen as in **Table 6**.

There are 125 combinations of input factors. With a sample size of 3, the total number of experiments is 375. The experimental results are collected. From the data, the optimized values of parameters are shown in **Figure 8**.

The GA model with DOE is run with the parameters defined by the experiment. The objective function is as in **Figure 9**.

The results of the GA model with DOE are shown as:

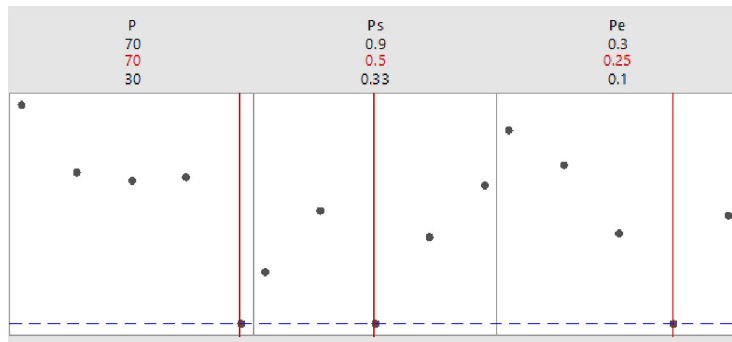
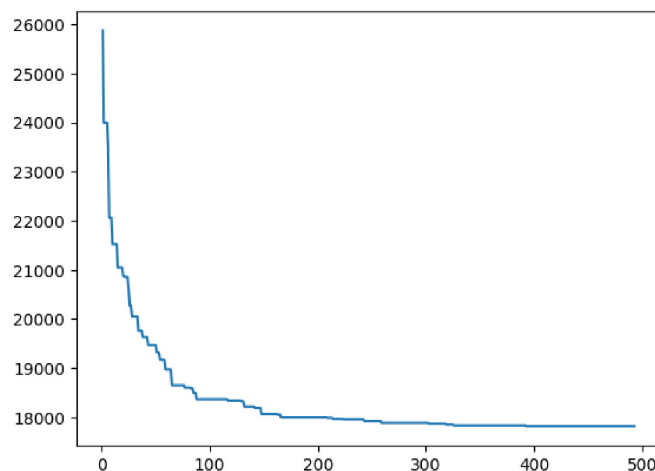
$$Q = \begin{vmatrix} 9 & 0 & 0 & 0 & 9 & 0 & 0 & 4 & 4 & 0 & 2 \\ 6 & 4 & 5 & 0 & 2 & 5 & 6 & 8 & 9 & 2 & 2 & 7 \\ 4 & 4 & 7 & 0 & 6 & 2 & 6 & 0 & 0 & 6 & 0 & 0 \end{vmatrix}$$

With objective value is 17,816.925.

Table 7 presents the comparison results between the current heuristic method, the GA algorithm, and the GA with DOE.

TABLE 7 | Comparison of results.

	The current method	The GA	The GA with DOE
Objective value	25,182	18,336	17,816
Improvement	-	27.18%	29.25%

**FIGURE 8** | The values of parameters to give the minimum objective value.**FIGURE 9** | The result of the objective function of the GA model with design of experiment (DOE).

Conclusion

The dynamic inventory planning problem has been modeled. The GA algorithm has been used to solve this mathematical model. The solution of the GA algorithm is better than the solution of the currently used empirical algorithm. However, it is still a suboptimal solution.

The experimental design is used to optimize the parameters of the GA algorithm. The solution of the GA algorithm with DOE is better than the solution of the GA algorithm. However, the algorithms are only used to solve the pilot problem with three items. The future development direction is for real problems with many items.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships

that could be construed as a potential conflict of interest.

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