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RESEARCH

# A bimodal supply chain game model for apparel enterprises considering consumer channel preferences

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Considering the consumer's channel preference, this paper studies the pricing strategies of clothing manufacturers and retailers mainly selling online on traditional e-commerce platforms and live e-commerce platforms, and constructs two sales modes: direct selling mode and resale mode. Based on the Stackelberg game model, the optimal pricing and profit of direct selling mode and resale mode under centralized decision-making are solved, and the variables such as imported discount, consumer trust, and demand satisfaction are analyzed in detail. The results show that a large discount can't guarantee the maximization of platform profits and a suitable discount can effectively improve the profits of retailers and manufacturers. Traditional platform retailers effectively improve consumers' trust in goods, which is conducive to improving their own and manufacturers' profits; When the cost paid by consumers when purchasing goods from different channels is different, the change of consumers' shopping cost will directly affect the pricing of goods by the two platforms.

**Keywords:** consumer channel preference, dual-channel supply chain, Stackelberg game, discount strength, consumer trust

## 1. Introduction

Many technologies such as online shopping software and mobile payment have changed consumers' consumption habits. Meanwhile, in the context of the epidemic, offline sales platforms are difficult to maintain, which has given rise to many clothing manufacturers mainly engaged in online sales to meet the market demand for products. Most of these enterprises choose to provide basic products to different platform retailers, and then the retailers sell them. A small number of clothing manufacturers hope to build their own brands, so they choose to enter third-party e-commerce platforms for sales. Before the emergence of live shopping, the domestic e-commerce market was divided by several major e-commerce platforms. However, the current pattern of China's e-commerce platforms is undergoing tremendous changes, among which the rise of Douyin live selling has had a profound impact on traditional e-commerce shopping platforms (such as Taobao and JD.com). This change is mainly due to the characteristics of live e-commerce platforms, such as low prices, time limits, and limited quantities, which bring more changes to the platform game. Douyin live selling refers to the anchors showing and recommending products to the audience through live broadcast on the Douyin platform, and providing real-time purchase functions. Unlike traditional e-commerce shopping, Douyin live selling has the following notable characteristics:

1. Enhanced interactivity: audiences can interact with anchors in real time during the live broadcast, asking questions, commenting, and liking, making the shopping process more engaging and entertaining.
2. Personalized recommendation: Douyin platform uses user portraits and algorithms to recommend personalized products and anchors according to users' interests and preferences, improving the accuracy of purchases.
3. Limited time promotion: shopping on live e-commerce platforms often adopts strategies such as low

price, limited time, and exclusive offers. Through cooperation with brand merchants, they provide relatively low prices and exclusive offers to attract users to participate in purchases. In addition, elements such as star anchors, product demonstrations, and real-time interactions are also widely used in live broadcasting. Traditional e-commerce platforms are also constantly innovating, but they place more emphasis on social factors such as user reviews and product ratings.

4. Shopping experience: traditional e-commerce platforms are mainly conducted through webpages or mobile apps. Usually, users independently choose products and place orders for purchase, with various payment methods such as online payment and cash on delivery. While shopping on live e-commerce platforms is carried out in the form of real-time live broadcast, users can watch anchors showing products and demonstrating usage methods in the live broadcast, emphasizing instant purchase. Anchors provide real-time purchase links during the live broadcast, and users can directly click to purchase to quickly snap up special limited-time products, while creating a sense of urgency and attraction.

These characteristics of live e-commerce platforms make Douyin live selling products quickly popular in a short time, and have impacted and revolutionized traditional shopping platforms. The mode of traditional shopping platforms mainly relies on consumers' independent choice, while Douyin live selling products is more in line with young people's consumption habits and shopping needs through real-time interaction and personalized recommendation. The characteristics of low-price, limited-time and limited sales strategy, personalized recommendation, and real-time interaction have become important factors to attract users.

Based on this background, this paper studies the pricing strategies of clothing manufacturers and retailers mainly engaged in online sales on traditional e-commerce platforms and live e-commerce platforms, considering the factor of consumer channel preference.

Different from the relationship between traditional e-commerce platforms and offline retail, in the past, consumers prefer to personally experience the fabric texture, color and design style of goods through offline physical channels. However, as shopping on live e-commerce platforms is carried out through real-time live broadcast, users can watch anchors showing products and demonstrating the use of methods in the live broadcast, and audiences can interact with anchors in real time and ask questions during the live broadcast, which to some extent combines the advantages of real-time experience of offline shopping and the advantages of cheap and convenient online shopping.

However, as live e-commerce platforms are based on the once-priced pricing of traditional e-commerce platforms, it is necessary to study how manufacturers and retailers determine the discount level and how to set parameters such as consumer channel preference and demand satisfaction when implementing low-price discounts on live e-commerce platforms, so as to determine the optimal price and improve the profitability of the entire supply chain. When making pricing decisions, whether it is traditional e-commerce platforms or live e-commerce platforms, the reputation of the platform itself, the cost of consumers purchasing goods from different channels, and the competitiveness between the two channels directly affect the setting of the retail price of goods. For live e-commerce platforms, the size of the discount rate also affects the pricing strategies of manufacturers and retailers. At the same time, in order to ensure the effectiveness of the dual-channel supply chain, manufacturers need to design reasonable coordination schemes to ensure the profits of retailers. Therefore, when implementing low-price promotional activities, live e-commerce platforms need to consider how to set the optimal discount rate to increase their profits, manufacturers need to consider how to set wholesale prices to maximize their profits and coordinate the profit relationship between the two platform retailers, and retailers need to set retail prices according to the wholesale prices provided by manufacturers.

According to existing research, preferential promotional activities of e-commerce platforms have a direct impact on the pricing strategies and profits of each member in the dual-channel supply chain (1, 2). Therefore, based on different sales models (direct sales model and resale model) under the dual-channel supply chain, this paper uses the Stackelberg game model to solve the optimal sales price of online retail channels under centralized and decentralized decision-making of manufacturers and retailers, and the reasonable range of discount rate generated by the second pricing of the live e-commerce platform compared with the primary pricing of the traditional platform when the live e-commerce platform implements time-limited low-price activities.

Closely related research areas of this paper include research on dual-channel supply chain model, research on channel preferential policies, and research on the coordination of dual-channel supply chain. The following literature will be sorted out and analyzed from these three aspects.

In terms of the impact and role of the manufacturer's dual-channel model, Chiang et al. (1) first found that the dual-channel sales model helps to reduce the damage of "double marginal effect" on the overall profit of the supply chain (3). Lu and Liu (3). Further proposed that the relative effectiveness of the network directly affects the profit of the manufacturer, and the low effectiveness of the network channel will directly reduce the profit of the manufacturer (4). Matsui et al. (4) showed that the pricing order of wholesale price and online direct sales price affects the overall profit of the dual-channel supply chain. Manufacturers can maximize profits by deciding online prices first and then wholesale prices (5). The emergence of retail giants has changed the market power structure. Li et al. (6) found that when small and medium-sized manufacturers open online channels, they are limited by strong contracts of retailers and have to obey the dominant power of strong retailers in deciding prices (6). Jafari et al. (7) compared the revenues of various members in the dual-channel supply chain model led by retailers under different game models, and found that retailer revenues are less affected by the game model and that the retail price is the highest when retailers and manufacturers cooperate (8). The above literature mainly studies the supply chain pricing strategy from the perspective of channel power structure, without considering the impact of channel competition strategies on the decisions of various members of the dual-channel supply chain.

In the dual-channel supply chain model, in order to improve the channel competitiveness, supply chain members begin to try to attract consumers through discounts and other ways. In terms of the research on channel discount coupons, scholars have conducted in-depth discussions on the impact of coupon issuing subjects and discount rates on consumer channel selection and supply chain pricing strategies and profits. For example, Dhar and Raju (9) constructed the impact model of cross-discount strategies between different products on consumer utility and obtained the cross-discount setting limits that can improve sales and profits (7). Hu et al. (10) compared the changes in the overall profit of the supply chain under the situation that different members of the supply chain launched discount coupons and found that the total revenue of the supply chain was the largest when the upstream and downstream members of the supply chain jointly launched discount coupons (11). These studies have verified the effectiveness of channel discount strategies. At the same time, some scholars have discussed the discount strength and the nature of applicable products. For example, Yang Guang et al. (12) studied the sales strategies of manufacturers under three discount levels of low, medium, and high in online channels and found that consumers' loss aversion preference and channel identification directly affect the revenue of manufacturers (12). Zhang Jiji et al. (14) compared and analyzed the impact of proportional discount and amount discount on supply chain profits and believed that for low-end products, proportional discount is more dominant than amount discount strategy; for high-end products, amount discount strategy is better (9). Venkatesan and Farris (13). analyzed the impact of customized discounts on consumer demand and manufacturer profits and found that customized discounts can enable retailers to obtain higher profits (14).

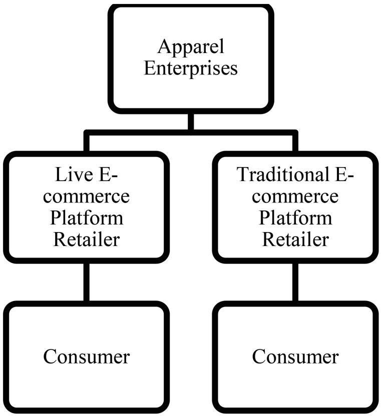
After the channel provides discounts, the original revenue structure of the supply chain will be changed. In order to ensure the overall revenue maximization of the supply chain without harming the interests of various channel members, profit coordination among members is needed. Commonly used coordination schemes include pricing coordination, revenue compensation coordination, revenue sharing coordination, etc. For example, Cattani et al. (15) tried to use a unified pricing strategy to ease the conflict between direct sales channels and physical channels and found that under certain conditions, a win-win situation can be formed (10). Zu Feng et al. (18) believed that the implementation of compensation policies by manufacturers can improve the pricing and revenue of various channels, and the amount of compensation and the degree of retailer's marketing efforts are related to the retailer's sales cost (16). Wang Wenbin et al. (20) established a hybrid sales supply chain model based on e-commerce platforms on the basis of previous studies and obtained a range of revenue sharing proportions that ensure that upstream and downstream members of the supply chain can profit from direct sales channels (17). Li et al. (18) established a dual-channel supply chain model consisting of manufacturers and retailers with different risk preferences and finally proposed that risk sharing contracts coordinate the benefits of supply chain members (19). Zhang Shen et al. (24) refined channel costs, considered the impact of platform deduction rate on the decision-making of dual-channel supply chain, and achieved coordinated pricing through the coordination pricing scheme of the Shapley value (13). Zhang Xuelong et al. (25) compared the coordination effects of various contract mechanisms such as revenue compensation and the Shapley value on the dual-channel supply chain with manufacturers playing a dominant role and concluded that the Shapley value method has higher coordination ability (15).

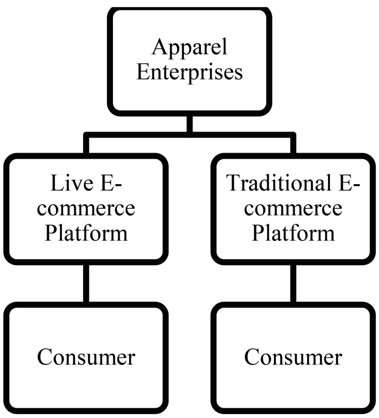
The existing dual-channel coordination research literature above mostly studies the dual-channel supply chain model combining online and offline. Although it is becoming more and more common for manufacturers to open direct sales channels through e-commerce platforms, there are few studies on manufacturers who mainly sell online in recent years. At the same time, the above studies generally classify traditional e-commerce platforms and live e-commerce platforms as one, without detailed analysis. In order to find a balance between the sales price of the two types of platforms, in order to seek to maximize the profit of the product. Based on the simple consumer price sensitivity coefficient, this paper solves the proportion of consumer demand that should be met by the live streaming e-commerce platform.

## 2. Model establishment and solution

### 2.1. Parameter description and model assumptions

In order to study the dual-channel pricing strategy of clothing manufacturing enterprises under the influence of consumer channel preference, this paper considers the situation that clothing enterprises mainly engaged in online sales sell clothes to consumers on live e-commerce platforms and traditional e-commerce platforms through resale and direct sales modes, and establishes the following two models:




In the resale mode, clothing manufacturing enterprises only act as suppliers and profit by supplying the same clothes to agents on different platforms. At the same time, the influence of consumer channel preference on consumers' purchasing channels is also considered. Assuming that the potential market demand is $q$, among which $\delta q$consumers accept both live e-commerce platforms and traditional e-commerce platforms, and the remaining $(1-\delta)\,q$ consumers refuse to buy clothes on live broadcasts and are only willing to buy clothes on traditional platforms (20).

At the same time, in order to describe the competitiveness of the two channels, the variable β is introduced as the channel competition coefficient, and considering the cost differences of the goods purchased on the two platforms, for example, the goods on the live shopping platform have the characteristics of time limit and limited quantity, which require consumers to stay in the live broadcast room to snap up. Missing the sales time or buying too slowly may lead to consumers failing to purchase the goods. Therefore, the cost variable $C$ is introduced to combine with β to measure consumers' choice of purchasing channels.

Different from offline shopping and live shopping, consumers shopping on the traditional platform means that they can only know the upper body effect and quality of the clothing after the purchase. Usually, in online shopping it is difficult to measure the quality of the goods through pictures. However, due to the anchor explanation and physical display of the live shopping, consumers' estimation of the value of the goods is more accurate and sufficient. Therefore, the variable $t$ is introduced as the consumer's trust in the traditional shopping platform (2, 4, 21)so $tv$ is the consumer's estimation of the value of the goods (22, 23).

Finally, considering the high price competitiveness of the goods on the live-streaming platform, a large number of goods will disturb the market order, and the retailers on the live-streaming platform are willing to adopt the strategy of hunger marketing, so the retailers on the platform will not fully meet the needs of consumers, but only meet$\gamma$ .

The relevant symbols and their meanings in the study are shown in the following table (18):

| Symbol | Meaning |
|---|---|
| q | Potential demand in the market |
| δ | Consumer channel preference |
| β | Channel competition coefficient |
| i | Two modes, i = 1,2 |
| $C_{si}$ | Cost paid by consumers shopping on the live-streaming platform |
| $C_{ti}$ | Cost paid by consumers shopping on the traditional platform |
| t | Consumers' trust in the traditional platform |
| v | Consumers' expected value of the product |
| $U_{si}$ | Consumers' utility of shopping on the live-streaming platform |
| $U_{ti}$ | Consumers' utility of shopping on the traditional platform |
| $P_{si}$ | Price of goods on the live-streaming platform |
| $P_{ti}$ | Price of goods on the traditional platform |
| $D_{si}$ | Market demand for goods on the live-streaming platform |
| $D_{ti}$ | Market demand for goods on the traditional platform |
| $\pi_i$ | Profit of clothing manufacturing enterprises |
| $w_i$ | Wholesale price of retailers |
| $c$ | Production cost of clothing manufacturing enterprises |
| $\pi_{si}$ | Profit of live-streaming platform |
| $\pi_{ti}$ | Profit of live-streaming platform |
| γ | Satisfaction degree of consumer demand on the live-streaming platform |

## 2.2. Demand function

The utility function of consumers on the live-streaming e-commerce platform is (16, 18)

$$U_{si} = v - P_{si} - \beta C_{si}$$

The utility function of consumers on traditional e-commerce platforms is:

$$U_{ti} = tv - P_{ti} - (1-\beta)\,C_{ti}$$

When $U_{s1} \geq U_{t1}$, $\beta \geq q\frac{(1-t)v+P_{ti}-P_{si}+C_{ti}}{C_{si}+C_{ti}}$, and at this time, consumers choose to buy on live-streaming e-commerce platforms. The market demand functions of different platforms are:

$$D_{si} = \delta q \frac{(1-t)\,v + P_{ti} - P_{si} + C_{ti}}{C_{si} + C_{ti}}$$

Different from the demand composition of live-streaming platforms, the demand composition of traditional e-commerce platforms is also composed of some consumers who are not willing to accept live-streaming shopping.

$$D_{ti} = \left(\delta q (1 - \frac{(1-t)v+P_{ti}-P_{si}+C_{ti}}{C_{si}+C_{ti}}) + (1-\delta)\,q\frac{tv-P_{ti}}{C_{ti}}\right)$$

Therefore, the demand function is as follows:

$$D_i = D_{ti} + D_{si}$$

## Resale model under decentralized decision-making

Under decentralized decision-making, the game model is composed of the relationship between the manufacturer and two retailers, and the Stackelberg game with the manufacturer as the leader and the two retailers as followers. Under this decision-making, only the situation where the manufacturer occupies the dominant position is considered. In the first stage, the manufacturer decides the platform's unified wholesale price$w_1$,and in the second stage, the two retailers decide the optimal retail price $P_{s1}$ and $P_{t1}$.

The objective function of the clothing manufacturing enterprise is:

$$\pi_1 = (w_1 - c)\,(D_{t1} + D_{s1})$$

The objective function of the live e-commerce platform is:

$$\pi_{s1} = (P_{s1} - w_1)\,\gamma D_{s1}$$

Considering that the price of the live e-commerce platform is more competitive than that of the traditional platform, the retailer adopts a hunger marketing strategy, which will not fully meet the needs of consumers, but only meet part of the needs$\gamma$ .

The objective function of the traditional e-commerce platform is:

$$\pi_{t1} = (P_{t1} - w_1)\,D_{t1}$$

Theorem 1: In the dual-channel supply chain, there exists a unique optimal equilibrium solution in the game between the clothing enterprise, the traditional platform retailer, and the live e-commerce platform retailer.

Proof

The model is solved by reverse induction.

Given $P_{t1}$, the first and second partial derivatives of $\pi_{s1}$ are obtained:

$$\frac{\partial \pi_{s1}}{\partial P_{s1}} = -\frac{\delta q \gamma\,(2P_{s1} - w_1 + (t-1)\,v - P_{t1} - C_{t1})}{C_{s1} + C_{t1}}$$

$$\frac{\partial^2 \pi_{s1}}{\partial P_{s1}^2} = -\frac{2\delta q \gamma}{C_{s1} + C_{t1}} < 0$$

According to $\frac{\partial^2 \pi_{s1}}{\partial P_{s1}^2} < 0$, it can be seen that there exists a unique equilibrium optimal pricing for live streaming platform retailers. If $\frac{\partial \pi_{s1}}{\partial P_{s1}} = 0$ is taken, it can be obtained:

$$P_{s1} = \frac{w_1 + (1-t)\,v + P_{t1} + C_{t1}}{2}$$

At this point, the formula is the strategy of the live platform retailer, that is, if the traditional platform retailer chooses $P_{t1}$, the live platform retailer will choose $P_{s1}$. In the first stage, when the traditional platform retailer chooses $P_{t1}$, it is predicted that the live platform retailer will act according to $P_{s1}\,(P_{t1})$. Therefore, the optimal solution problem of the traditional platform retailer is to choose $P_{t1}$ so that $\pi_{t1}\,(P_{t1}, P_{s1}\,(P_{t1}))$ is maximized.

The first and second partial derivatives of $\pi_{t1}$ are obtained:

$$\frac{\partial \pi_{t1}}{\partial P_{t1}} = \delta q (1 - \frac{P_{t1} + (1-t)\,v - P_{s1} + C_{t1}}{C_{s1} + C_{t1}}) + (-\frac{\delta q}{C_{s1} + C_{t1}} - \frac{(1-\delta)\,q}{C_{t1}})\,(P_{t1} - w_1) + (1-\delta)\,q\frac{tv - P_{t1}}{C_{t1}}$$

$$\frac{\partial^2 \pi_{t1}}{\partial P_{t1}^2} = \frac{2q\,(C_{s1}\,(\delta - 1) - C_{t1})}{C_{t1}\,(C_{s1} + C_{t1})} < 0$$

According to $\frac{\partial^2 \pi_{t1}}{\partial P_{t1}^2} < 0$, it can be seen that there is a unique optimal pricing equilibrium for the traditional platform retailer. Taking $\frac{\partial \pi_{t1}}{\partial P_{t1}} = 0$:

$$P_{t1} = \frac{(\delta C_{s1} - C_{t1} - C_{s1})\,w_1 + ((\delta C_{s1} - C_{t1} - C_{s1})\,t + \delta C_{t1})v - \delta\,(C_{t1}P_{s1} + C_{s1}C_{t1})}{2\delta C_{s1} - 2C_{t1} - 2C_{s1}}$$

Substituting $P_{s1}$ into $P_{t1}$ The optimal wholesale price at this time is:

$$P_{t1} = \frac{2w_1\,(C_{s1}\delta - C_{t1} - C_{s1}) + 2v\,((C_{s1}\delta - C_{t1} - C_{s1})\,t + C_{t1}\delta) - \delta C_{t1}\,(2C_{s1} + w_1 + (1-t)\,v + C_{t1})}{4C_{s1}\delta - 4C_{t1} - 4C_{s1} + C_{t1}\delta}$$

Simultaneously, the equilibrium optimal pricing for retailers on the live streaming platform is:

$$P_{s1} = \frac{(w_1 + (1-t)\,v + C_{t1})\,(2C_{s1}\delta - 2C_{s1} - 2C_{t1}) + w_1(C_{s1}\delta - C_{t1} - C_{s1}) + v\,((C_{s1}\delta - C_{t1} - C_{s1})\,t + C_{t1}\delta) - \delta C_{t1}C_{s1}}{4C_{s1}\delta - 4C_{t1} - 4C_{s1} + C_{t1}\delta}$$

Similarly, taking the second partial derivative of manufacturer profit with respect to wholesale price $w_1$ results in:

$$\frac{\partial^2 \pi_1}{\partial w_1^2} = \frac{2q(\delta-1)(C_{t1}(\delta+2)+2C_{s1}(1-\delta))}{C_{t1}(4C_{s1}(1-\delta)+C_{t1}(4-\delta))} < 0$$

Therefore, the optimal wholesale price at this point is:

$$w_1^* = \frac{-\left((2C_{s1}\delta^2-(2C_{t1}+4C_{s1})\delta+2C_{t1}+2C_{s1})t- C_{s1}\delta^2+C_{t1}\delta\right)v-\left((2C_{s1}-C_{t1})c+C_{t1}^2-2C_{s1}C_{t1}\right)\delta^2}{(2C_{t1}-4C_{s1})\delta^2+(2C_{t1}+8C_{s1})\delta-4C_{t1}-4C_{s1}}$$

$$+\frac{-\left((-C_{t1}-4C_{s1})c+2C_{t1}^2+2C_{s1}C_{t1}\right)\delta-2c(C_{s1}+C_{t1})}{(2C_{t1}-4C_{s1})\delta^2+(2C_{t1}+8C_{s1})\delta-4C_{t1}-4C_{s1}}$$

Substituting $w_1^*$ into $P_{s1}$ and $P_{t1}$ yields $P_{t1}^*$ and $P_{s1}^*$, which proves Theorem 1.

Theorem 2: As consumer trust $t$ in traditional shopping platforms increases, the optimal equilibrium wholesale price $w_1^*$ also increases, leading to higher profits $\pi_1$ for apparel manufacturing enterprises. Simultaneously, the pricing $P_{t1}^*$ by traditional platform retailers increases, with profits $\pi_{t1}$ initially decreasing and then increasing. In contrast, the pricing $P_{s1}^*$ on live shopping platforms decreases, resulting in lower profits $\pi_{s1}$.

Proof

To simplify the proof, let's represent the three optimal equilibrium prices as follows:

$$P_{t1}^* = \frac{2vt(C_{s1}\delta-C_{t1}-C_{s1})-\delta C_{t1}(1-t)v}{4C_{s1}\delta-4C_{t1}-4C_{s1}+C_{t1}\delta}+\varphi_{t1}$$

$$P_{s1}^* = \frac{((1-t)v)(2C_{s1}\delta-2C_{s1}-2C_{t1}) + vt(C_{s1}\delta-C_{t1}-C_{s1})}{4C_{s1}\delta-4C_{t1}-4C_{s1}+C_{t1}\delta}+\varphi_{s1}$$

$$w_1^* = \frac{-(2C_{s1}\delta^2-(2C_{t1}+4C_{s1})\delta+2C_{t1}+2C_{s1})tv}{(2C_{t1}-4C_{s1})\delta^2+(2C_{t1}+8C_{s1})\delta-4C_{t1}-4C_{s1}}+\varphi_1$$

where $\varphi_{t1}, \varphi_{s1}, \varphi_1$ denote the parts of the functions unrelated to $t$

$$\frac{\partial w_1^*}{\partial t} = \frac{-2v(C_{s1}\delta^2-(C_{t1}+2C_{s1})\delta+C_{t1}+C_{s1})}{(2C_{t1}-4C_{s1})\delta^2+(2C_{t1}+8C_{s1})\delta-4C_{t1}-4C_{s1}} > 0$$

$$\frac{\partial P_{t1}^*}{\partial t} = \frac{2v(-C_{t1}+C_{s1}(\delta-1))+v\delta C_{t1}}{4C_{s1}\delta-4C_{t1}-4C_{s1}+C_{t1}\delta} > 0$$

$$\frac{\partial P_{s1}^*}{\partial t} = \frac{(-C_{s1}\delta+C_{t1}+C_{s1})v}{4C_{s1}\delta-4C_{t1}-4C_{s1}+C_{t1}\delta} > 0$$

$$\frac{\partial \pi_1}{\partial t} = \frac{\partial w_1^*}{\partial t}D_1+\frac{(1-\delta)q(w_1^*-c)}{C_{ti}}\left(v-\frac{\partial P_{t1}^*}{\partial t}\right)$$

Here, $v-\frac{\partial P_{t1}^*}{\partial t} = \frac{2vC_{t1}(\delta-1)-2vC_{s1}}{4C_{s1}\delta-4C_{t1}-4C_{s1}+C_{t1}\delta} > 0$, hence $\frac{\partial \pi_1}{\partial t} > 0$

$$\frac{\partial \pi_{s1}}{\partial t} = \left(\frac{\partial P_{s1}^*}{\partial t}-\frac{\partial w_1^*}{\partial t}\right)\gamma D_{s1}+(P_{s1}^*-w_1^*) \gamma\delta q\frac{C_{t1}}{C_{s1}+C_{t1}}\left(-v+\frac{\partial P_{t1}^*}{\partial t}-\frac{\partial P_{s1}^*}{\partial t}\right)$$

Also, $-v+\frac{\partial P_{t1}^*}{\partial t}-\frac{\partial P_{s1}^*}{\partial t} = \frac{vC_{t1}+vC_{s1}(1-\delta)}{4C_{s1}\delta-4C_{t1}-4C_{s1}+C_{t1}\delta} < 0$, leading to $\frac{\partial \pi_{s1}}{\partial t} < 0$

$$\frac{\partial \pi_{t1}}{\partial t} = \left(\frac{\partial P_{t1}^*}{\partial t}-\frac{\partial w_1^*}{\partial t}\right)D_{t1}+(P_{t1}^*-w_1^*)\frac{\partial D_{t1}}{\partial t}$$

In conclusion, $\frac{\partial w_1^*}{\partial t} > 0$, $\frac{\partial P_{t1}^*}{\partial t} > 0$, $\frac{\partial P_{s1}^*}{\partial t} < 0$, $\frac{\partial \pi_1}{\partial t} > 0$, $\frac{\partial \pi_{s1}}{\partial t} < 0$,and $\frac{\partial \pi_{t1}}{\partial t}$ initially decreases, becoming positive when \(t\) reaches a certain point. Thus, the second theorem is proved.

This proposition elucidates the relationship between the optimal equilibrium game solution of supply chain members and consumer trust in traditional shopping platforms. Higher trust brings higher pricing and profits for apparel manufacturing enterprises and traditional platform retailers. However, for live platform retailers, it suggests avoiding high trust product competition with traditional platform retailers, instead focusing on promoting and selling low-trust products.

Theorem 3: Reasonable secondary pricing by live shopping platform retailers is advantageous for market share capture.

Proof

As the pricing by live shopping platform retailers is a secondary pricing after the completion of primary pricing on traditional platforms, the game between them can be considered a complete information dynamic game with the following relationship:

$$P_{s1} = \theta P_{t1}$$

At this point, the profit for live shopping platform retailers is:

$$\pi_{s1} = (\theta P_{t1}-w_1)\gamma\delta q\frac{(1-t)v+P_{t1}-\theta P_{t1}+C_{t1}}{C_{s1}+C_{t1}}$$

Taking partial derivatives:

$$\frac{\partial \pi_{s1}}{\partial \theta} = \frac{-P_{t1}\delta q\gamma\cdot(2P_{t1}\theta-w_1+(t-1)v-P_{t1}-C_{t1})}{C_{s1}+C_{t1}}$$

$$\frac{\partial^2 \pi_{s1}}{\partial \theta^2} = -\frac{-P_{t1}^2\delta q\gamma}{C_{s1}+C_{t1}} < 0$$

As the discount intensity increases, the profit of live shopping platform retailers initially rises and then falls. The optimal discount rate is:

$$\theta^* = \frac{w_1+(1-t)v+P_{t1}+C_{t1}}{2P_{t1}}$$

Thus, within a certain range, offering discounts is beneficial for the revenue of retailers.

For traditional platform retailers, aware that live shopping platform retailers will perform secondary pricing based on their primary pricing, the profit function is:

$$\pi_{t1} = (P_{t1} - w_1)\left(\delta q\left(1 - \frac{(1-t)v + P_{t1} - \theta P_{t1} + C_{t1}}{C_{s1} + C_{t1}}\right) + (1-\delta)\, q\frac{tv - P_{t1}}{C_{t1}}\right)$$

To illustrate the impact of live platform discounts on traditional platform retailer profits:

$$\frac{\partial \pi_{t1}}{\partial \theta} = \frac{P_{t1}\delta q(P_{t1} - w_1)}{C_{s1} + C_{t1}}$$

Since traditional platform retailers set prices $P_{t1}$ higher than the wholesale price $w_1$, live platform discounts are bound to reduce traditional platform retailer profits.

This theorem highlights discount promotions as a common marketing strategy, emphasizing that, to a certain extent, lowering prices can enhance retailer revenue. However, excessive price reduction may lead to a price war, causing market disruption and harm to both sides.

Theorem 4: If live shopping platform retailers aim to attract more potential consumers by reducing profits through increased discount intensity while ensuring non-negative profits, the discount intensity should satisfy$\theta \in \left(\max\left(\frac{w_1}{P_{t1}}, \frac{(1-t)v + P_{t1} - C_{s1}}{P_{t1}}\right), 1\right)$.

Proof

The function related to the discount intensity $\theta$ for live shopping platform retailers is:

$$D_{s1} = \delta q\frac{(1-t)v + P_{t1} - \theta P_{t1} + C_{t1}}{C_{s1} + C_{t1}}$$

$$\pi_{s1} = (\theta P_{t1} - w_1)\gamma\delta q\frac{(1-t)v + P_{t1} - \theta P_{t1} + C_{t1}}{C_{s1} + C_{t1}}$$

When consumers prefer both platforms and choose the live shopping platform when both are willing, we have:

$$\theta = \frac{(1-t)v + P_{t1} - C_{s1}}{P_{t1}}$$

When the profit of live shopping platform is zero:

$$\theta = \frac{w_1}{P_{t1}}$$

Thus, $\theta \in \left(\max\left(\frac{w_1}{P_{t1}}, \frac{(1-t)v + P_{t1} - C_{s1}}{P_{t1}}\right), 1\right)$ proving the fourth theorem.

This theorem indicates that live shopping platform retailers managing multiple categories of products can attract more potential consumers by lowering prices for certain categories, even at the cost of reduced profits. Combining with the second theorem, live platform retailers can strategically lower prices for traditional e-commerce platform products with high consumer trust, avoiding excessive investment in high-trust products and potentially gaining a larger market with minimal losses.

Theorem 5: Based on the cost differences for consumers purchasing goods from different platforms, when $w_1 > \frac{v(2+t)}{3}$, for traditional platform retailers, changes in consumer shopping costs have a smaller impact on their own channel retail price than on the opposite side's retail price. If, at this point, consumer channel preference ($\delta$) is within $\delta \in \left(0, \frac{2}{3}\right)$, then live shopping platform retailers also adhere to this theorem.

Proof

$$R_s = \frac{\partial P_{s1}}{\partial C_{s1}} - \frac{\partial P_{t1}}{\partial C_{s1}} = -\frac{C_{t1}\delta(1-\delta)(vt + 2v - 3w_1) + \delta^2 C_{t1}^2 + 2\delta C_{t1}^2}{((4\delta - 4)C_{s1} + C_{t1}\delta - 4C_{t1})^2}$$

$$R_t = \frac{\partial P_{t1}}{\partial C_{t1}} - \frac{\partial P_{s1}}{\partial C_{t1}} = -\frac{C_{s1}^2\left(12\delta^2 - 20\delta + 8\right) + C_{t1}^2\left(\delta^2 - 6\delta + 8\right) + C_{s1}C_{t1}\left(8\delta^2 - 24\delta + 16\right) + C_{s1}\delta(1-\delta)(2v + vt - 3w_1)}{((4\delta - 4)C_{s1} + C_{t1}\delta - 4C_{t1})^2}$$

For $R_t = \frac{\partial P_{t1}}{\partial C_{t1}} - \frac{\partial P_{s1}}{\partial C_{t1}}$,

when $\delta \in \left(0, \frac{2}{3}\right)$,$12\delta^2 - 20\delta + 8 > 0$,

and when $\delta \in (0, 1)$,$8\delta^2 - 24\delta + 16 > 0$, $\delta^2 - 6\delta + 8 > 0$

Thus, when $w_1 < \frac{v(2+t)}{3}$,$R_t = \frac{\partial P_{t1}}{\partial C_{t1}} - \frac{\partial P_{s1}}{\partial C_{t1}} < 0$

For traditional platform retailers, when $w_1 = \frac{v(2+t)}{3}$ and $\delta \in \left(0, \frac{2}{3}\right)$, $R_t = \frac{\partial P_{t1}}{\partial C_{t1}} - \frac{\partial P_{s1}}{\partial C_{t1}} < 0$. This implies that $P_{t1}$ changes at a slower rate with $C_{t1}$ than $P_{s1}$ changes with$C_{t1}$. For live shopping platform retailers, when $w_1 < \frac{v(2+t)}{3}$, $R_s = \frac{\partial P_{s1}}{\partial C_{s1}} - \frac{\partial P_{t1}}{\partial C_{s1}} < 0$, indicating that $P_{s1}$ changes at a slower rate with $C_{s1}$ than $P_{t1}$ changes with $C_{s1}$. Thus, the fifth theorem is proved.

This theorem elucidates the relationship between the costs consumers incur when purchasing goods through traditional and live shopping platforms and the retail prices of goods on different platforms. In specific circumstances, both live platform retailers and traditional platform retailers experience a smaller impact on their own channel prices from changes in the cost consumers bear when purchasing goods through their channels compared to the impact on the opposite channel's price.

## 2.4. Centralized decision-making marketing model

In this decision-making model, apparel manufacturers directly join both live e-commerce platforms and traditional e-commerce platforms for sales. Therefore, in this model, the objective function for apparel manufacturers is given by:

$$\pi_2 = P_{t2}D_{t2} + P_{s2}\gamma D_{s2}$$

which can be expressed as:

$$\pi_2 = P_{t2}\left(\delta q\left(1 - \frac{(1-t)\,v + P_{t2} - \theta P_{t2} + C_{t2}}{C_{s2} + C_{t2}}\right) + (1-\delta)\,q\frac{tv - P_{t2}}{C_{t2}}\right) + \theta P_{t2}\gamma\delta q\frac{(1-t)\,v + P_{t2} - \theta P_{t2} + C_{t2}}{C_{s2} + C_{t2}}$$

Under this decision-making mode, only the optimal equilibrium discount rate for live e-commerce platforms is considered to maximize the total profit for the apparel manufacturing enterprise without further distinguishing the profit differences between the two channels.

Theorem 6: Under centralized decision-making, there exists a unique optimal equilibrium discount rate $\theta$ and optimal equilibrium pricing $P_{t2}$, maximizing the overall profit for the apparel manufacturing enterprise.

Proof

The profit$\pi_2$ for the apparel manufacturing enterprise with respect to the discount rate $\theta$ has first- and second-order partial derivatives:

$$\frac{\partial\pi_2}{\partial\theta} = -\frac{P_{t2}\delta q(2P_{t2}\gamma\theta + \left((t-1)v - P_{t2} - C_{t2}\right)\gamma - P_{t2})}{C_{s2} + C_{t2}}$$

$$\frac{\partial^2\pi_2}{\partial\theta^2} = -\frac{2P_{t2}^2\gamma\delta q}{C_{s2} + C_{t2}} = 0$$

This indicates the existence of an optimal equilibrium discount rate that maximizes the profit for the apparel manufacturing enterprise. When $\frac{\partial\pi_2}{\partial\theta} = 0$, the optimal equilibrium discount rate is obtained:

$$\theta = \frac{\left((1-t)\,v + P_{t2} + C_{t2}\right)\gamma + P_{t2}}{2\gamma P_{t2}}$$

Based on the Stackelberg game model, the secondary pricing behavior of the apparel manufacturing enterprise on the live e-commerce platform transforms the overall optimization problem into $\pi_2(P_{t2}, \theta P_{t2}(\theta))$.

Therefore, taking the first and second partial derivatives of the profit function of the apparel manufacturing firm (denoted as $\pi_2$) with respect to the pricing set by the traditional e-commerce platform ($P_{t2}$):

$$\frac{\partial\pi_2}{\partial P_{t2}} = \frac{q\left((C_{t2}\delta\gamma^2 - C_{t2}\delta)P_{t2} + \left((C_{t2}\delta - C_{t2}\delta t)v + C_{t2}^2\delta\right)\gamma^2 + \left((C_{t2} + C_{s2})\delta - 2C_{t2} - 2C_{s2}\right)\gamma + C_{t2}\delta\right)}{2C_{t2}\gamma(C_{s2} + C_{t2})}$$

$$\frac{\partial^2\pi_2}{\partial P_{t2}^2} = \frac{\delta q(\gamma^2 - 1)}{2\gamma(C_{s2} + C_{t2})} < 0$$

Hence, there exists an optimal equilibrium price, $P_{t2}$, that maximizes the profit of the apparel manufacturing firm. Setting$\frac{\partial\pi_2}{\partial P_{t2}} = 0$ yields:

$$\mathbf{P_{t2}} = \frac{\left((\mathbf{C_{t2}}\delta\mathbf{t} - \mathbf{C_{t2}}\delta)\mathbf{v} - \mathbf{C_{t2}^2}\delta\right)\gamma^2 + \left((-\mathbf{C_{t2}} - 2\mathbf{C_{s2}})\delta + 2\mathbf{C_{t2}} + 2\mathbf{C_{s2}}\right)\gamma - \mathbf{C_{t2}}\delta}{\mathbf{C_{t2}}\delta\gamma^2 - \mathbf{C_{t2}}\delta}$$

## 2.5. Model extension

Due to the marketing strategy of apparel manufacturers on live e-commerce platforms, not all consumers opting to purchase goods on these platforms will be satisfied. Only a proportion represented by $\gamma$will be catered to. However, the previously described model assumes that the quantity $\delta q$ of consumers will inevitably choose to buy goods, overlooking the market demand changes due to discount pricing on live e-commerce platforms. For ease of calculation, we assume the price sensitivity function of market demand is given by:

$$\sigma = (1-\gamma)$$

Theorem 7: There exists an optimal equilibrium demand fulfillment, maximizing the profit for the apparel manufacturing enterprise.

Proof

In this model assumption, the profit function for the apparel manufacturing enterprise is given by:

$$\pi_2 = P_{t2}\left(\delta q\sigma\left(1 - \frac{(1-t)\,v + P_{t2} - P_{s2} + C_{t2}}{C_{s2} + C_{t2}}\right) + (1-\delta)\,q\sigma\frac{tv - P_{t2}}{C_{t2}}\right) + P_{s2}\gamma\delta q\sigma\frac{(1-t)\,v + P_{t2} - P_{s2} + C_{t2}}{C_{s2} + C_{t2}}$$

Taking the first and second derivatives with respect to $\gamma$:

$$\frac{\partial\pi_2}{\partial\gamma} = -\frac{P_{s2}\delta q\left((1-t)\,v + P_{t2} - P_{s2} + C_{t2}\right)(1-2\gamma)}{C_{s2} + C_{t2}} - P_{t2}\left(\delta q\left(1 - \frac{(1-t)v + P_{t2} - P_{s2} + C_{t2}}{C_{s2} + C_{t2}}\right) + (1-\delta)q\sigma\frac{tv - P_{t2}}{C_{t2}}\right)$$

$$\frac{\partial^2\pi_2}{\partial\gamma^2} = \frac{2P_{s2}\delta q\left((t-1)v + P_{s2} - P_{t2} - C_{t2}\right)}{C_{s2} + C_{t2}} < 0$$

Thus, there exists an optimal equilibrium demand fulfillment, maximizing the profit for the apparel manufacturing enterprise. Setting $\frac{\partial\pi_2}{\partial\gamma} = 0$ :

$$\gamma = \frac{\left(\left((C_{s2}P_{t2} - C_{t2}P_{s2})\delta - (C_{s2} + C_{t2})P_{t2}\right)t - (C_{t2}P_{t2} + C_{s2}P_{s2})\delta\right)v + \left(C_{s2}P_{t2}^2 + C_{s2}C_{t2}P_{t2} + C_{t2} + C_{t2}^2P_{s2}\right)\delta - (C_{s2} + C_{t2})P_{t2}^2}{2C_{t2}P_{s2}\delta\left(v(t-1) + P_{s2} - C_{t2} - P_{t2}\right)}$$

As the chosen price sensitivity function is a linear function, and real-world market demand and prices often exhibit a non-linear relationship, this model may not perfectly reflect the impact of demand fulfillment on the profit of the apparel manufacturing enterprise. However, it generally suggests that an appropriate level of demand fulfillment can enhance the profit of the apparel manufacturing enterprise.

## 3. Conclusion

Consumer trust in traditional e-commerce platforms positively influences the wholesale prices and profits of apparel manufacturers, as well as the retail prices of live e-commerce platform retailers. Conversely, it negatively impacts the retail prices and profits of traditional platform retailers. In other words, an increase in consumer trust in traditional e-commerce platforms leads to higher wholesale prices for apparel manufacturers and higher retail prices for traditional e-commerce platform retailers, while the retail prices of live e-commerce platform retailers will decrease. For live e-commerce platform retailers, products with low trust on traditional e-commerce platforms are typically those with stronger consumer desires, high-end fabric, or sophisticated designs. Retailers should capitalize on this characteristic of certain apparel products, reinforce consumer attitudes toward products with low trust on traditional e-commerce platforms, and leverage live shopping interactions and displays to highlight the features of live e-commerce platforms, making it easier to enhance overall company profits. Simultaneously, for high-trust products, live platform retailers can adopt a strategy of small profits and high sales. Under the premise of no losses, they can set appropriate discount levels to exchange minimal profits for higher consumer popularity and customer loyalty. For manufacturers and traditional platform retailers, brand promotion and increasing product stickiness can continuously build and enhance consumer trust in products, serving as an additional growth point for seeking business development.

If there are differences in the costs incurred by consumers when purchasing goods from traditional e-commerce platforms and live e-commerce platforms, changes in consumer shopping costs will directly impact the pricing of goods on both platforms. For traditional e-commerce platforms, when $w_i = \frac{v(2+t)}{3}$ and $\delta \in \left(0, \frac{2}{3}\right)$ with the shopping costs of live e-commerce platforms remaining constant, the increase in shopping costs for traditional e-commerce platforms has a smaller impact on the retail prices of traditional platform retailers than on those of live platform retailers. Similarly, when $w_i = \frac{v(2+t)}{3}$ and the shopping costs of traditional e-commerce platforms remain constant, the increase in shopping costs for live e-commerce platforms has a smaller impact on the retail prices of live platform retailers than on those of traditional platform retailers. Therefore, regardless of which channel's retailer is setting retail prices, they should not only consider changes in the shopping costs of the other channel but also pay close attention to changes in their own channel's shopping costs. In other words, for a dual-channel supply chain game, it is crucial not only to consider changes in one's own channel value but also, more importantly, to focus on subtle improvements in the competitive channel. Changes in the prices and shopping costs of the competitive channel are closely related to consumer purchasing decisions. Timely insight into these changes and quicker formulation of response strategies are effective ways to address the consumer market in the game of dual-channel supply chains.

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