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RESEARCH

Visualization of information technologies in the teaching of mathematics

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The main purpose of educational tools in mathematics is to check the results with quality. There are many different tools that help students develop logical thinking through interactive activities. Mathematics education tools are created for innovative, interactive, and dynamic learning in various mathematical disciplines. The goal is to show the importance of applying information technologies (ITs) with the intention of realizing the principle of visualization. Data visualization is a key tool in online learning that helps students better understand complex ideas while providing teachers and administrators with insight into the educational process. It also makes it easier for teachers to monitor student progress and identify where additional information or support is needed. The use of visual elements in teaching can increase students' interest and help them get involved in the learning process, which is the primary goal of teaching—to put the student in the center of the educational process. According to the theoretical and practical knowledge collected so far, it has been confirmed that the use of ITs in mathematics classes leads to the achievement of useful knowledge among students. Also, using visualization, it is possible to improve students' arithmetic, algebraic, and geometric intuitions.

Keywords: visualization, mathematics education, innovations in mathematics education, mathematical software GeoGebra, positive effects of visualization

Introduction

Modern teaching includes a wide variety of multimedia materials that enhance the quality of instruction, boost motivation, improve subject comprehension, and promote superior individual advancement of individuals in accordance with their intellectual capabilities. Participants may improve their existing knowledge and apply it in practical ways with the use of information and communication technologies, notably electronic learning tools and technologies. since current teaching methods emphasize putting theoretical knowledge into practice.

The educational process is one of the most important jobs. Apart from learning itself, it also includes education, which is an important part of society. Modern education is marked by the use of various means, as well as the application of various methods that allow for daily improvement and innovation. The teaching of mathematics follows technological development and is suitable for the use of various educational tools, and it changes in order to approach complex topics for students in the simplest way. Learning through play and interesting activities can motivate students and help them understand the material better, because such knowledge is more long-lasting, and mathematics teaching makes it possible. By using computers

in mathematics lessons, students can acquire new knowledge and strengthen existing knowledge. The modern educational approach requires the teacher to be skilled in working with technology in order to improve his methodical work.

In the introduction to mathematics teaching, there is often a need for a visual presentation of mathematical objects so that it would be easier for students to understand them. Traditional teaching requires a lot of didactic and teaching material, while information technologies (ITs), as part of modern teaching, offer numerous possibilities. They allow multiple examples to be presented in one lesson, and students can return to the same lesson several times, which significantly helps in easier and faster mastering of the material. In mathematics teaching, we can recognize two types of visualization: symbolic and objective. In this regard, ITs can be used to create two-dimensional shapes and three-dimensional simulations. Symbolic visualization is realized by means of pictures, drawings, graphics, and schemes, while objective visualization involves models of geometric figures. The use of ITs enriches the content of classes, and students become more motivated, which is crucial for the acquisition of effective knowledge. Many abstract mathematical structures will be clearer and easier for students to understand if the principle of visualization through IT is applied.

All teachers aim to provide quality teaching that includes a variety of methods, interesting ideas, clear rules, a pleasant working atmosphere, and an appropriate level of expertise. It is important that there is good cooperation between teachers and students on a professional and personal level. Students should be active in order to contribute to the development of their personality. Teaching methods are strategies that teachers use to organize lessons and achieve their goals. There is no single method that can satisfy all objectives. Since each student has his own way of learning, the methods must be varied and combined. It is necessary to continuously evaluate which methods are useful and which are not in order to improve the quality of teaching. Basic methods in the field of informatics/computer science are: (1) method of oral presentation, (2) method of demonstration, (3) method of conversation, (4) method of practical work, and (5) method of reading and working with the text. So that the teaching would be different and that lectures and demonstrations would not prevail, and the use of other methods would be enabled.

Visualization in mathematics teaching

Visualization proves to be a powerful tool in learning mathematics because it can turn abstract ideas into concrete experiences (1). With carefully designed visual support, mathematics becomes engaging, intuitive, and easier to understand, opening learning opportunities that encourage creativity, exploration, and the development of critical thinking in students. With this approach, students get a chance to master mathematics more deeply and see it as an important and interesting field.

Modern computer technology significantly affects mathematics education, including its content, methods, techniques, curricula, and organization of teaching and learning processes (2). In modern mathematics education, it is no longer a question of whether to use computer technology, but it is important to find the best solutions for how and in what way to apply it in the classroom in order to improve the quality of teaching and make the learning process more efficient.

The use of technology in education and learning must be carefully planned, with a well-considered approach, appropriate theoretical foundations, and in accordance with the specifics of the subject. Technology should be used in a broad and responsible way in order to improve the process of learning mathematics. The role of the teacher in planning, designing, and carrying out activities that use computers is very important. Teachers are responsible for determining when a computer can enhance learning opportunities and what type of technology should be used to achieve lesson objectives. Modern technologies allow teachers to better adapt their teaching methods to the needs of students and to offer creative options that support the learning process and the acquisition of knowledge and skills (3). Computers in the learning process can be very useful for students when calculating, making graphs, working with larger data sets, navigating the symbolic world, connecting various representations of mathematical objects, performing experiments, formulating assumptions, and testing their accuracy. Computer technology allows students to focus more on thinking about mathematical concepts and exploring and discovering mathematical knowledge. With the support of computers, students have the opportunity to develop and demonstrate a deeper understanding of mathematical ideas and can engage in more advanced mathematical topics than would be possible in a regular classroom.

The ability of computing technologies to connect mathematical concepts with visual representations in a way that stimulates mathematical thinking and understanding is very important for learning and teaching mathematics. Today, modern educational technology has almost no limits when it comes to the visual representation of any mathematical or real objects, as well as adapting these representations to the understanding of students through the senses and the learning process. “When objects from everyday life become displayed on the screen, they lose concrete form, while mathematical entities that are usually abstract gain concreteness on the screen.” Presenting mathematical concepts within a multimedia environment opens up opportunities for activities that encourage new

ideas and thinking, which motivates students to actively participate, think independently, explore concepts and their connections, solve different problems, create new information, and ask new questions.

Technology can visualize learning processes, which can encourage more complex cognitive activities, as well as support thinking, visual expression of ideas, and creative, non-linear (divergent) thinking (4). In the educational space, there is a wide range of software tools that can be used in teaching mathematics. Most specialized math programs, such as dynamic geometry software and computer algebra systems (CAS), allow symbolic, numerical, and graphical operations to be performed, creating space for active exploration of mathematical structures through various representations. Programs, for computer algebra CAS, allow performing numerical calculations and manipulating symbolic mathematical expressions. Most of those programs also support the visualization of some algebraic objects, while the user cannot directly change their graphical representation (5). Some of the well-known software packages include Mathematica, Maple, Derive, MatLab, and Scientific WorkPlace. These tools can be used to cover topics such as algebra, mathematical analysis, analytic geometry and numerical mathematics, and the like.

Dynamic geometry software is particularly important in visualizations and constructions that remain mathematically consistent even after object changes (6). The interactive environment of these software tools makes it possible to carry out high-quality experimental research in the teaching of geometry and in the processing of functions. Dynamic visualization allows students to see mathematical aspects that are often too difficult or impossible to demonstrate using only paper and pencil. It is difficult to imagine how we would display dynamic processes without the use of a computer, and the time required to achieve the same goal would be significantly longer. This increases the effectiveness of mathematics teaching. Also, new ways are opened for the introduction of modern mathematical content, highlighting the visual characteristics of existing programs and adjusting the tasks used in their processing.

Modern teaching of mathematics relies on didactic principles that are interconnected. In the initial teaching of mathematics, the principle of obviousness has a special importance, since basic mathematical concepts are formed in that period. The opinion of students in primary education is still at a concrete level, which means that in order to acquire mathematical concepts, children first need to use representations and then abstractions. This principle implies more than acquiring knowledge through perception; it also includes the use of previously acquired performances. During the 1980s, the importance of visual processing and interest in the role of visual representation in mathematics education grew rapidly.

Research dealing with visualization in mathematics started gradually during the eighties of the last century. During that period, the difficulties and advantages of visual thinking in mathematics were analyzed (7–11). The prevailing belief was that visual thinking could be considered the ability to switch between different ways of thinking when faced with a particular problem, and the education system at the time was subject to criticism for insufficient use of visualization and development of visual thinking skills. Sommer (12) stated that schools are directly responsible for the decline of visual thinking, pointing out that many teachers do not want to apply visualization in teaching processes, even when it is natural and possible. Also, he stated that teachers who work with older students often consider the visualization of mathematical material as something childish and primitive. Subjects such as technical drawing, art and other subjects requiring spatial orientation were often considered less important and did not require serious intellectual commitment.

During the nineties, technology and the use of computers became more and more present in teaching activities related to school mathematics. The importance of visual representations of mathematical content, including diagrams, tables, and graphs, was recognized in addition to already existing algebraic representations. In recent times, the development of technology has led to a significant impact of computers, laptops, the Internet, tablets, and mobile phones on the learning style and work habits of students. During the 2000s, the research was expanded to include the semiotic aspects of visualization, focusing on didactics, that is, methods of learning mathematics in order to improve the application and effectiveness of visualization.

Innovations in mathematics education - GeoGebra

Modern teaching techniques emphasize the practical application of theoretical knowledge in real-world contexts. With the aid of computers, interactive resources offer a unique method of instruction that may be used in any field, but notably in the natural and technical sciences, where it is particularly useful. The ability to picture different operations is crucial. One of the primary objectives of mathematics education is to empower students to reason critically and overcome challenges in all areas of their lives. In this case, the emphasis is specifically on how theoretical knowledge is applied in a practical setting. The four stages of problem solving, according to mathematician George Polya, are problem identification, planning, plan execution, and back analysis (13). Therefore, we may conclude that Polya's problem-solving approach is well suited to inquiry-oriented instruction. The usage of interactive material may be readily incorporated into this paradigm.

Mathematics education software is designed to provide a creative and engaging method for teaching various

branches of mathematics. The free, dynamic application GeoGebra integrates geometry, algebra, and analysis. Markus Hohenwarter created this curriculum at Florida Atlantic University with the goal of enhancing the mathematics learning experience between high school and college.

As a dynamic geometric tool, GeoGebra enables the creation of constructions using points, vectors, dimensions, lines, and sections, as well as graphical representation of functions with the possibility of dynamic changes. It also provides calculus functions, such as finding key points of functions (zero, extrema, inflection points), entering equations and coordinates directly, and calculating derivatives and integrals. Because of these functionalities, GeoGebra is an excellent tool for multiple representations of mathematical entities (14). It can be especially useful for students in solving mathematical challenges both in and out of class.

GeoGebra is designed to make it easier for students to understand mathematics. Students can easily manipulate variables by dragging "free" objects around the desktop or using sliders. With this method of manipulating free objects, students can observe how changes are reflected in the behavior of dependent objects. This enables the research of mathematical relations through dynamic problem solving.

GeoGebra also provides great opportunities for cooperative learning. This type of learning is an excellent approach for many areas of mathematics. Classical lectures should be replaced by topic-oriented interactive sessions. The main role of the teacher is not only to teach or explain mathematics individually but also to create situations that help students to develop the necessary mental constructs more quickly. In this context, GeoGebra offers various opportunities for collaborative learning, including working in small groups, interactive lessons for the whole group, or even individual or group student presentations.

On the one hand, GeoGebra provides teachers with important options for creating interactive educational content that can be made available online. GeoGebra encourages teachers to apply technology in teaching in the areas of visualization of mathematics, "research" in mathematics, interactive lessons via the website or at a distance, various ways of applying mathematics, etc.

Key features of GeoGebra

GeoGebra is an interactive mathematical software intended for primary and secondary schools, which combines geometry, algebra, analysis, tables, and statistics. The name of this program combines parts of the words "geometry" and "algebra." Although its development was spearheaded by Markus Hohenwarter, numerous professionals in the fields of education, mathematics, and computer science have helped to advance the software. at the GeoGebra Institute's sites throughout the world. It is positioned as dynamic geometry software, which means that it provides opportunities for interactive work in geometry. In addition to classic elements such as combinations and intersections of lines and circles, drawing of ellipses, hyperbolas, and parabolas is also available. On the other hand, GeoGebra allows the input of coordinates and equations, which are then interpreted geometrically. It is possible to set conic sections and lines using explicit and implicit equations, and lines can also be defined parametrically. GeoGebra supports working with Cartesian and polar coordinates. As this software handles numbers, angles, vectors, points, lines, and conic sections, it can also be considered a CAS tool. The numerical functions of this program provide a variety of geometric commands: centroids, foci, vertices, principal axes, and diameters of conic sections, as well as direction coefficients, direction vectors, line normal vectors, and many others. Graphical, algebraic, and tabular parts of the system are interconnected and dynamic.

Between the user and GeoGebra there is communication that takes place through the interface. This interface can be presented in different formats, known as perspectives. To learn and display analytical geometry in the plane, it is most convenient to use the perspective, which consists of two main parts—the algebraic and the geometric window. The algebra window contains the coordinates of the points and the equations of the curves to be controlled, as well as all other numerical values, such as angle sizes, lengths, and areas. This window can be visible or hidden. The geometry window is similar to other dynamic geometry software (DGS). In it, figures can be drawn with the right mouse button or with the help of the input field. After the figures are created, the user can move them with the mouse or keyboard. Algebraic expressions that follow changes in the geometry window are visible in the algebra window. The main purpose of this software, which is education, has affected the way of data entry. Entering data in natural notation, almost identical to writing in a notebook or on a blackboard, attracts the attention of students and teachers and makes working with GeoGebra easier.

GeoGebra is developed using Java, which allows its use regardless of the operating system, be it Windows, Linux, MacOS, or Unix. In addition, it can be used through a web browser. There are also versions for iPad, Windows, and Android tablets. If the program is installed on a computer, it can be used regardless of whether the computer is connected to the Internet. GeoGebra allows direct printing of drawn diagrams and can display dynamic drawings or construction descriptions as web pages, as well as drawing surfaces as images. In GeoGebra version 5, there is a possibility to create new tools and customize the tool palette. Conditional text display helps when creating lesson visualization worksheets, allowing teachers to highlight specific parts of the content at a given time.

Positive effects of visualization in mathematics teaching

If students remain passive and unmotivated during class, they will not be able to develop the necessary understanding, draw conclusions, or apply their knowledge and skills towards educational goals. Therefore, it is of great importance to introduce into the class teaching methods and techniques that will encourage active learning, enabling students to be as engaged and actively participate as possible. These techniques enable communication between students and teachers, as well as between students (15). It improves intellectual engagement that leads not only to the acquisition of new information but also the development of other important student skills. Research-based teaching is one of the most effective ways to encourage student engagement. In this type of teaching, students independently explore and create new knowledge, building their own concepts, which helps them remember what they have learned longer (16). Easier connection of material with other topics enables better application of knowledge, and the feelings that arise during the process of discovery increase curiosity and internal motivation. In addition to increased curiosity, research also encourages creativity, which is developed by devising solutions and asking questions during the process; critical thinking that manifests itself in the analysis of methods and results; as well as the development of abstract thinking; the ability to work with a lack of information; self-regulation; communication; and taking responsibility when collaborating with other students during research. As a result of all these skills and traits that students acquire during research classes, they will gain a clearer picture of math and science, and this can further motivate students to enroll in science and math programs in high school or college (17). All these qualities and skills are extremely important for students to be successful in accordance with what today's society expects of us as individuals.

Conclusion

The use of ITs during teaching enables and motivates students to predict outcomes and to imagine mathematical concepts while developing their mental images. Using computers helps students notice changes when task conditions change, making it easier to test assumptions. Technology can be most successfully applied to support students in gathering information, taking tests, making adjustments, and deciding which assumptions to retain or discard as they consider these mathematical concepts and explore mathematics. Virtual and physical manipulatives have proven to be very useful learning tools when used with diverse groups of students in a variety of educational settings. Using multiple views and being able to switch between different representations makes the learning process easier for students and can contribute to deeper understanding. In parallel with the development of technology, teachers should improve their knowledge about it so that its application is effective and appropriate (18). By using information and communication technologies, students are enabled to better understand and acquire mathematical content. Namely, the implementation of IT technologies includes visualization, which can potentially facilitate students' understanding of abstract structures. Many mathematical concepts can become easier to understand through images in 2D or 3D format, which further encourages students to acquire new knowledge.

Visual learning enables the natural connection of abstract ideas with concrete examples (19). This involves combining scientific and easy-to-understand ways of presenting content. It also combines individual and group work methods, as well as solving problems related to development. Teachers can approach their work creatively and proactively, taking into account personal and age-related characteristics such as perception, memory, and thinking. In addition, by organizing work on the creation of visual aids, students' interests and cognitive abilities develop.

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Conflict of interest

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