BOHR Journal of Computational Intelligence
and Communication Network
2024, Vol. 2, No. 1, pp. 1–4
DOI: 10.54646/bjcicn.2024.11
www.bohrpub.com

RESEARCH

Linear solution of the passive localization problem with combined single-station direction finding and two-station time difference information

Tao Yu*

China Academy of Management Science, Beijing, China

***Correspondence:**
Tao Yu,
tyt0803@163.com

Received: 24 August 2023; **Accepted:** 19 September 2023; **Published:** 10 January 2024

Based on the main station's direction measurement value and the time difference measurement value of the two stations, two definite solution equations can be listed in the polar coordinate system by directly utilizing the path difference equation and the cosine theorem, and the analytical solution can be obtained by solving the two definite solution equations jointly. Further comparison of the ranging errors shows that the ranging accuracy of the two-station directional and time-difference positioning system is only slightly worse than that of the three-station time-difference positioning system. The new results provide a better basis for the engineering design of two-station hybrid localization.

Keywords: time-of-arrival, direction of arrival, master-slave station, direction finding, dual-station time difference, dual-station passive localization, ranging, passive localization and tracking

1. Introduction

The authors have recently read literature on the problem of dual-site passive localization of targets with joint direction of arrival (DOA) and time difference of arrival (TDOA) information (1–5), and the author's research objective is to solve the problem of passive localization by integrating dual-site TDOA measurement information when only one site has directional capability. The authors' original intuition for this localization method was that an analytical solution should exist, but after reviewing the available literature, it was found that all the solutions given were non-linear. Some of these papers directly show that passive localization of moving targets by combining DOA and TDOA information is essentially a non-linear estimation problem (6).

To this end, the authors asked GPT3.5 the following question: In planar two-station passive localization of a target with combined DOA and TDOA information, where only one station performs a direction finding, is there an analytical method for such localization? Answer from GPT3.5: In planar two-station passive localization, where only one station performs the direction finding, the analytical method for localizing a target with combined angle and time difference information is not available. There is no resolving method for localization. This is because in planar two-station passive positioning, at least two stations are needed to determine the position of the target, and when only one station performs the direction finding, we cannot determine the arc in which the target is located, so we cannot determine the position of the target from the direction finding information. The time difference information can only determine the hypersphere where the target location is located, and cannot determine the specific location of the target. Therefore, it is not possible to perform planar two-station passive localization when only one station implements the direction finding. If target localization is required, it is necessary to add another receiving site or use other localization methods.

In fact, based on planar analytic geometry, when the number of equations in the system is greater than or equal to the number of unknown variables, the system will have a unique analytic solution. In the case of two-station localization, where the baseline between the two stations is known, there are really only two unknown variables: the radial distances from the two probe stations to the target. When only single-station direction finding is used, or when only two-station time difference measurements are used, the conditions for obtaining an analytic solution really do not exist, because only one fixed equation can be listed. However, when the above two measurements are used in combination, two fixed-solution equations can be listed in the polar coordinate system by directly utilizing the path difference equation and the cosine theorem, thus satisfying the analytical condition that the number of equations is greater than or equal to the number of unknown variables. It is thus theoretically possible to give analytical results.

In this article, based on the corollary made in polar coordinate system, the authors obtain the ranging analytical solution of the two-station passive positioning method with joint information of angle and time difference.

2. DEDUCE

As shown in **Figure 1**, the direction-measuring and time-difference passive localization system consists of a joint master station and a slave station. Master station A can use detection equipment to determine the arrival azimuth of the radiation source signal, and time difference information can be obtained between stations A and B, regardless of the time difference measurement method used.

In fact, if the geometry containing the target and the two receiving stations is considered as a triangle, the baseline already established between the two stations, based on the theory of plane geometry, determines the base of the fixed triangle, the orientation determines an angle in a triangle between a hypotenuse and a base side, and the time difference measurement gives the difference between the two hypotenuses.

The equation of the path difference obtained from the time-difference measurements:

$$\Delta r_{12} = r_1 - r_2 = v_c \Delta t_{12} \tag{1}$$

Using the cosine theorem, this can be obtained:

$$r_2^2 = r_1^2 + d^2 - 2dr_1 \cos\theta \tag{2}$$

Deform equation (1):

$$r_2 = r_1 - v_c \Delta t_{12} \tag{3}$$

Substituting into equation (2) obtained from the cosine theorem:

$$(r_1 - v_c \Delta t_{12})^2 = r_1^2 + d^2 - 2dr_1 \cos\theta(4)$$

The final ranging solution can be obtained:

$$r_1 = \frac{d^2 - (v_c \Delta t_{12})^2}{2\left(d\cos\theta - v_c \Delta t_{12}\right)} \tag{4}$$

Using the MATLAB, the relative computational errors at different baseline lengths are given in **Figure 2**, thus indicating that the derived formulae are correct.

The values of the parameters used in the calculations are:$r_1{=}600km$, $d{=}50km$ when not specified.

3. Positioning accuracy analysis

3.1. Hybrid dual station

The relative ranging error is analyzed by total differential method. First setup:

$$r_1 = \frac{P}{Q}$$

$$P = d^2 - (v_c \Delta t_{12})^2$$

$$Q = 2\left(d\cos\theta - v_c \Delta t_{12}\right)$$

A. Ranging error from θ

$$\frac{\partial r_1}{\partial \theta} = \frac{1}{Q^2}\left(Q\frac{\partial P}{\partial \theta} - P\frac{\partial Q}{\partial \theta}\right)$$

$$\frac{\partial P}{\partial \theta} = 0$$

$$\frac{\partial Q}{\partial \theta} = -2d\sin\theta$$

B. Ranging errors from time-differential measurements

$$P = d^2 - (v_c \Delta t_{12})^2$$

$$Q = 2\left(d\cos\theta - v_c \Delta t_{12}\right)$$

$$\frac{\partial r_1}{\partial \Delta t_{12}} = \frac{1}{Q^2}\left(Q\frac{\partial P}{\partial \Delta t_{12}} - P\frac{\partial Q}{\partial \Delta t_{12}}\right)$$

$$\frac{\partial P}{\partial \Delta t_{12}} = -2v_c^2 \Delta t_{12}$$

$$\frac{\partial Q}{\partial \Delta t_{12}} = -2v_c$$

C. Simulations

When the errors of each observation are zero-mean and independent of each other, for a two-station system, there is a relative ranging error:

$$\sigma_r = \frac{1}{r_1}\left[\left|\frac{\partial r_1}{\partial \Delta t_{12}}\right|\sigma_{\Delta t} + \left|\frac{\partial r_1}{\partial \theta}\right|\sigma_\theta\right] \tag{5}$$

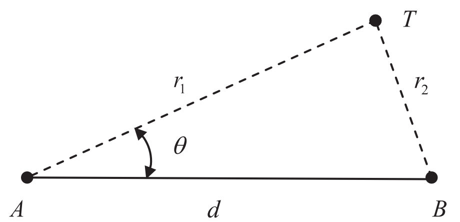

FIGURE 1 | Principle of dual-station localization.

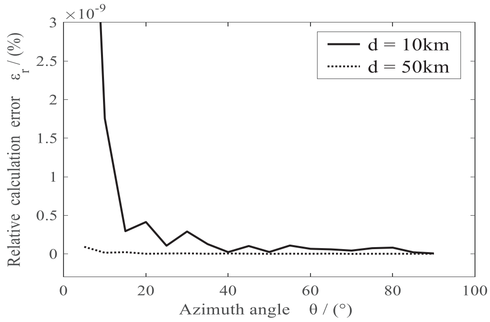

FIGURE 2 | Relative computational errors for different baseline lengths.

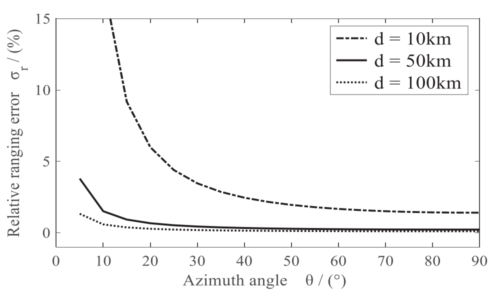

FIGURE 3 | Relative ranging error at different baselines.

where: $\sigma_{\Delta t}$ and σ_θ are the root-mean-square errors of the time difference and angle measurement errors, respectively.

The values of the parameters used in the calculations when not specified:

$$r_1 = 600km, d = 50km, \sigma_\theta = 0.5^0, \sigma_{\Delta t} = 100ns.$$

Using the MATLAB, the relative ranging errors at different baseline lengths are given in **Figure 3**. Obviously, a long baseline is good for reducing the error. **Figure 4** gives the relative ranging error at different radial distances; the closer the target distance, the better the localization accuracy.

3.2. Comparison with three stations with time difference

It has been shown that if the path difference equation and the geometric auxiliary relation are combined, a linear solution of the planar dual-base three-station localization equation can be obtained (7). For the three-station time-difference

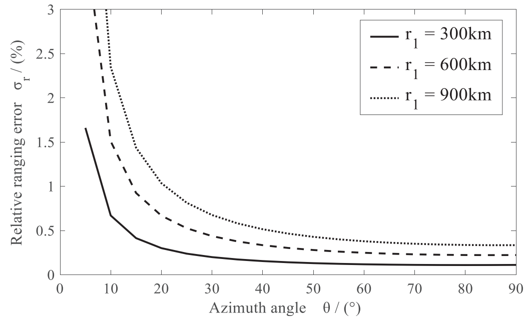

FIGURE 4 | Relative ranging error at different radial distances.

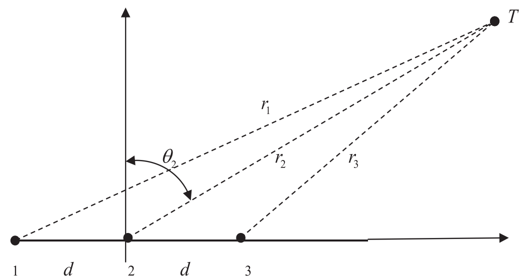

FIGURE 5 | One-dimensional double base array.

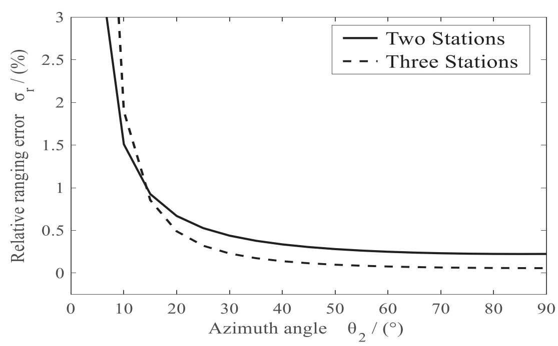

FIGURE 6 | Comparison of relative ranging errors.

passive localization system shown in **Figure 5**, there is a linear analytical solution:

$$r_2 = \frac{2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2}{2\left(\Delta r_{12} - \Delta r_{23}\right)} \tag{6}$$

Setting:

$$r_2 = \frac{P_2}{Q_2}$$

$$P_2 = 2d^2 - \Delta r_{12}^2 - \Delta r_{23}^2$$

$$Q_2 = 2\left(\Delta r_{12} - \Delta r_{23}\right)$$

A. Ranging error from time difference Δt_{12} measurements

$$\frac{\partial r_2}{\partial \Delta t_{12}} = \frac{1}{Q_2^2}\left(Q_2 \frac{\partial P_2}{\partial \Delta t_{12}} - P_2 \frac{\partial Q_2}{\partial \Delta t_{12}}\right)$$

$$\frac{\partial P_2}{\partial \Delta t_{12}} = -2\Delta r_{12}\frac{\partial \Delta r_{12}}{\partial \Delta t_{12}}$$

$$\frac{\partial Q}{\partial \Delta t_{12}} = 2\frac{\partial \Delta r_{12}}{\partial \Delta t_{12}}$$

$$\frac{\partial \Delta r_{12}}{\partial \Delta t_{12}} = v_c$$

B. Ranging error from time difference $_{\Delta t_{23}}$ measurements

$$\frac{\partial r_2}{\partial \Delta t_{23}} = \frac{1}{Q_2^2}\left(Q_2\frac{\partial P_2}{\partial \Delta t_{23}} - P_2\frac{\partial Q_2}{\partial \Delta t_{23}}\right)$$

$$\frac{\partial P_2}{\partial \Delta t_{23}} = -2\Delta r_{23}\frac{\partial \Delta r_{23}}{\partial \Delta t_{23}}$$

$$\frac{\partial Q}{\partial \Delta t_{23}} = -2\frac{\partial \Delta r_{23}}{\partial \Delta t_{23}}$$

$$\frac{\partial \Delta r_{23}}{\partial \Delta t_{23}} = v_c$$

3.3. Simulation calculation

When the errors of the individual observations are all zero-mean and independent of each other, for a three-station system, there is a relative ranging error:

$$\sigma_{r2} = \frac{1}{r_2}\left[\left|\frac{\partial r_2}{\partial \Delta t_{12}}\right|\sigma_{\Delta t} + \left|\frac{\partial r_2}{\partial \Delta t_{23}}\right|\sigma_{\Delta t}\right] \tag{7}$$

The parameter values used in the calculations are the same as in the previous section. By using MATLAB, **Figure 6** compares the relative ranging errors of the two positioning systems. It can be seen that the accuracy of the three-station time-difference localization system is slightly better than that of the two-station.

4. Conclusion

This thesis corrects the existing misconception that dual-station passive localization with joint angle and time difference is a non-linear estimation problem, based on the fundamental theorem of plane analytic geometry, and gives analytic results by using the time-difference equations and trigonometric relations directly in the polar coordinate system.

The corollary of this article, as well as many of the authors' previous studies, shows that certain problems that do not have an analytic solution in the Cartesian coordinate system are able to obtain an analytic solution when transferred to the polar coordinate system. It is very confusing why many researchers analyze problems only in the Cartesian coordinate system. It is even more puzzling why people fail to see the basic fact that, based on mathematical common sense, at least two definite equations can be formulated based on the two types of measurements and that there are only two unknowns in the plane from which it is possible to obtain definite results.

The results in this article lay a new foundation for the application of hybrid two-station, and it is likely that the existing complex filtering algorithms will be simplified and handled (8). Although the hybrid two-station measurement approach has a larger localization error than the three-station time-difference localization system, the hybrid two-station system is much superior because the sites are easier to find and the system is more economically configured (9, 10).

References

1. Xiong J, Xiong J. An improved algorithm for joint multistation AOA/TDOA passive localization closure form. *Telecommun Technol.* (2019) 59:672–7.
2. Guo F. A fixed single-station passive radar tracking method based on TOA and DOA. *J Terahertz Sci Electron Inform.* (2015) 13:908–12.
3. Cao J, Wang X, Cheng S, et al. Dual-base radar localization method based on TDOA and AOA. *Fire Command Control.* (2015) 40:10–2.
4. Deng P, Li L, Fan PC. A hybrid TDOA/AOA localization algorithm and its performance analysis. *J Radio Wave Sci.* (2002):633–6. doi: 10.13443/j.cjors.2002.06.018
5. Tian K, Wang J, Wan S. Passive localization and tracking of master-slave station based on TOA and DOA measurements of motion radiation sources. *J Air Force Radar Coll.* (2001):23–5.
6. Wang D, Lin S, Li C. Dual-station approximate unbiased localization algorithm based on angle and time difference. *Radar Sci Technol.* (2008):371–7.
7. Tao Y. *Technology of Passive Detection Location.* Beijing: National Defense Industry Press (2017).
8. Li S, Deng H. A dual-station direction finding/time difference joint localization algorithm based on EKF. *Radio Commun Technol.* (2015) 41:38–40+93.
9. Yan C, Wang Y. Accuracy analysis of two-station azimuthal time difference positioning system. *Radio Commun Technol.* (2011) 37:44–6.
10. Pan Chengger. *Research on Joint Localization Method for Passive System Direction Finding and Time Difference Frequency Difference.* MS Thesis. Xi'an: Xi'an University of Electronic Science and Technology (2005).